Conceptual Questions

1. When a third polarizer is inserted at 45° between two orthogonal polarizers, some light is transmitted. If, instead of a single polarizer at 45°, we insert a large number ***N*** of polarizers, each time rotating the axis of polarization over an angle 90°/***N***,
   a) no light   b) less light   c) the same amount of light   d) more light
   gets through. [1]

   ANS: D

   Since each successive filter is only slightly rotated to the one previous, only a very small amount of light will be absorbed. As ***N*** gets larger and larger, the amount absorbed gets less and less, and approaches complete transmission (see below). With one filter at a 45° angle, it blocks half the amount of light that impinges on it, so it lets less light through.

   Input interpretation:

plot	$\cos^2\left(\frac{\pi}{2x}\right)^x$	$x = 0$ to 100

   Plot:

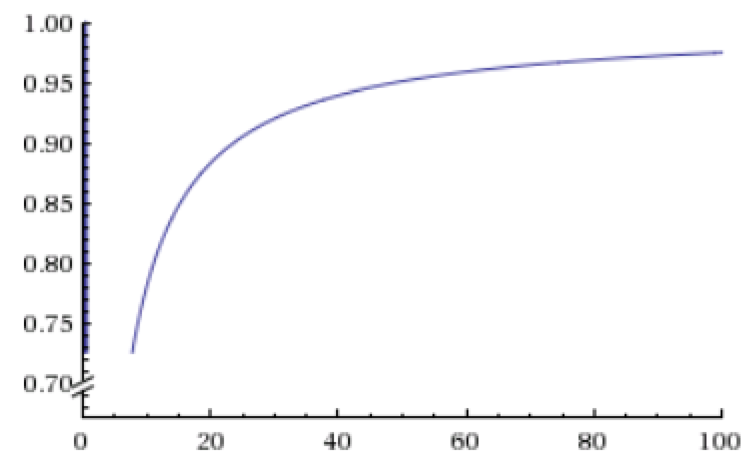

Image created at wolframalpha.com

2. Explain the advantage of polarized sunglasses over normal tinted sunglasses. [2]

   Polarized sunglasses completely (100%) block horizontally polarized glare and block all other polarizations of light 50%. Regular sunglasses just block 50-75% of all light coming in. The advantage of polarized sunglasses is the total elimination of glare. Even if regular sunglasses block a glare at 75%, the glare is so intense that it still makes it difficult for our eyes.

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[1] Peer Instruction – A User's Guide, Mazur, Optics CT 4
[2] Physics 6th Edition, Giancoli, Chapter 24 Questions, #28

3. If the Earth's atmosphere were 50 times denser than it is, would sunlight still be white, or would it be some other colour? Explain. [3]

If the atmosphere were 50% more dense, sunlight would be much redder than it is now. As the atmosphere increased in density, more and more of the blue light would be scattered away in all directions, making the light that reaches the ground very red. Think of the color of a deep red sunset, but this would be the color even at noon.

4. a) Does a beam of infrared photons always have less energy than a beam of ultraviolet electrons? Explain.
b) Does a single infrared photon always have less energy than a single ultraviolet photon? Explain.[4]

(*a*) No. It is possible that you could have many more IR photons in that beam than you have UV photons in the other beam. In this instance, even though each UV photon has more energy than each IR photon, the IR beam could be carrying more total energy than the UV beam.
(*b*) Yes. A single IR photon will always have less energy than a single UV photon due to the inverse relationship between wavelength and frequency (or energy) of light. The long wavelength IR photon has a lower frequency and less energy than the short wavelength UV photon.

5. Is it possible for the de Broglie wavelength of a "particle" to be greater than the dimensions of the particle? To be smaller? Is there any direct connection? [5]

It is possible for the de Broglie wavelength $(\lambda = h/p)$ of a particle to be bigger than the dimension of the particle. If the particle has a very small mass and a slow speed (like a low-energy electron or proton) then the wavelength may be larger than the dimension of the particle.

It is also possible for the de Broglie wavelength of a particle to be smaller than the dimension of the particle if it has a large momentum and a moderate speed (like a baseball). There is no direct connection between the size of a particle and the size of the de Broglie wavelength of a particle. For example, you could also make the wavelength of a proton much smaller than the size of the proton by making it go very fast.

Problems

6. Calculate the percentage of light travelling through two crossed polarizing filters if the angle between the polarizing directions is
a) 10° b) 30° c) 70° d) 85° [6]

a) $I_2 = 0.5 I_o \cos^2 \theta$
$I_2 = 0.5 I_o \cos^2 10°$
$I_2 = 0.485 I_o$
$\frac{I_2}{I_o} = 48.5\,\%$

b) $\frac{I_2}{I_o} = 37.5\,\%$

c) $\frac{I_2}{I_o} = 5.85\,\%$

d) $\frac{I_2}{I_o} = 0.380\,\%$

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[3] Physics 6th Edition, Giancoli, Chapter 24 Questions, #32
[4] Physics 6th Edition, Giancoli, Chapter 27 Questions, #26
[5] Physics 6th Edition, Giancoli, Chapter 27 Questions, #20
[6] Physics Book Two, Irwin Publishing, Chapter 10 Problems, #60

7. At what angle should two polarizing filters be positioned to reduce the intensity of light by 60°? [7]

$$I_2 = 0.4I_o$$
$$I_2 = 0.5I_o \cos^2\theta$$
$$0.4I_o = 0.5I_o \cos^2\theta$$
$$\theta = \cos^{-1}\sqrt{\frac{0.4}{0.5}}$$
$$\theta = 26.6°$$

8. Three polarizing filters are placed on top of one another. If the angle between the first two filters is 60° and the angle between the first and third filter is 70°, find the percentage of light exiting the last polarizing filter. [8]

$$I_2 = 0.5I_o \cos^2\theta_1 \text{ and } I_3 = I_2\cos^2\theta_2$$
$$I_3 = 0.5I_o\cos^2\theta_1\cos^2\theta_2$$
$$I_3 = 0.5I_o\cos^2 60°\cos^2 10°$$
$$I_3 = 0.121I_o$$
$$\frac{I_3}{I_o} = 12.1\%$$

9. Unpolarized light passes through five successive Polaroid sheetsm each of whose axis makes a 45° angle with the previous one. What is the intensity of the transmitted beam, in terms of the original intensity ***I_0***? [9]

When plane-polarized light hits a sheet oriented at an angle θ,

$$I_2 = I_1\cos^2\theta.$$

For $\theta = 45°$,

$$\frac{I_2}{I_1} = \cos^2 45° = \tfrac{1}{2}.$$

So sheets two through five will each reduce the intensity by $\frac{1}{2}$.

Since the first sheet will reduce the intensity of the unpolarized incident light by $\frac{1}{2}$ as well, the intensity of the transmitted beam will be

$$I = I_0\left(\tfrac{1}{2}\right)^5 = \boxed{0.031I_0.}$$

[7] Physics Book Two, Irwin Publishing, Chapter 10 Problems, #61
[8] Physics Book Two, Irwin Publishing, Chapter 10 Problems, #62
[9] Physics 6th Edition, Giancoli, Chapter 24 Problems, #61

10. A 2.0-W laser emits a coherent light beam at a wavelength of 632.4 nm. Assuming that all the power is radiated, how many photons leave the laser tube every second? [10]

$P = 2\ \text{W}$, $\lambda = 632.4\ \text{nm} = 6.324 \times 10^{-7}\ \text{m}$

We are to find the number of photons leaving the laser tube per second. Let us symbolize this quantity by N_γ.

Using Planck's equation, we can express the energy for a single photon:

$$E_\gamma = \frac{hc}{\lambda}$$

The number of photons leaving the tube can be found as follows:

$$N_\gamma = \frac{P}{E_\gamma}$$

$$N_\gamma = \frac{P\lambda}{hc}$$

$$N_\gamma = \frac{(2\ \text{W})(6.324 \times 10^{-7}\ \text{m})}{(6.626 \times 10^{-34}\ \text{J}\cdot\text{s})(3.0 \times 10^{8}\ \text{m/s})}$$

$$N_\gamma = 6.36 \times 10^{18}\ \text{photons/s}$$

11. The human eye can respond to as little as 10^{-18} J of light energy. For a wavelength at the peak of visual sensitivity (550 nm), how many photons lead to an observable flash? [11]

The energy of a single photon is

$$E = hf = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34}\ \text{J}\cdot\text{s})(3.00 \times 10^{8}\ \text{m/s})}{(550 \times 10^{-9}\ \text{m})} = 3.62 \times 10^{-19}\ \text{J}.$$

Therefore, the number of photons required for an observable flash is calculated as

$$\frac{10^{-18}\ \text{J}}{3.62 \times 10^{-19}\ \text{J}} = 27.7.$$

Rounding up, we find that $\boxed{28}$ photons are required.

12. A photon has a wavelength of 400 pm.
   a) What is its frequency?

   $\lambda = 400\ \text{pm} = 4.0 \times 10^{-10}\ \text{m}$

   a) The frequency of the photon can be found using the wave equation:

   $$f = \frac{c}{\lambda}$$

   $$f = \frac{3.0 \times 10^{8}\ \text{m/s}}{4.0 \times 10^{-10}\ \text{m}}$$

   $$f = 7.5 \times 10^{17}\ \text{Hz}$$

   b) What is its momentum?

   b) The momentum of the photon can be computed using de Broglie's equation:

   $$p = \frac{h}{\lambda}$$

   $$p = \frac{6.626 \times 10^{-34}\ \text{J}\cdot\text{s}}{4.0 \times 10^{-10}\ \text{m}}$$

   $$p = 1.66 \times 10^{-24}\ \text{N}\cdot\text{s}$$

[10] Physics Book Two, Irwin Publishing, Chapter 12 Problems, #22
[11] Physics 6th Edition, Giancoli, Chapter 27 Problems, #16

c) What is its mass equivalence? [12]

c) The mass equivalence can be found using de Broglie's equation:

$$p = mv$$

$$m = \frac{p}{c}$$

$$m = \frac{1.66 \times 10^{-24}\ \text{N}\cdot\text{s}}{3.0 \times 10^{8}\ \text{m/s}}$$

$$m = 5.53 \times 10^{-33}\ \text{kg}$$

13. An electric stove produces many infrared photons. If the peak wavelength of the radiation coming from the stove element is 10µm, what is the momentum of the released photons? [13]

$\lambda = 10\ \mu\text{m} = 1 \times 10^{-5}\ \text{m}$

Using de Broglie's equation:

$$p = \frac{h}{\lambda}$$

$$p = \frac{6.626 \times 10^{-34}\ \text{J}\cdot\text{s}}{1 \times 10^{-5}\ \text{m}}$$

$$p = 6.63 \times 10^{-29}\ \text{N}\cdot\text{s}$$

14. An electron at rest is struck be an x-ray photon. If the scatter angle is 180° and the final speed of the electron is 7.12×10^5 m/s, what was the wavelength of the incident photon? [14]

$\theta = 180°$, $v_f = 7.12 \times 10^5\ \text{m/s}$

From the conservation of energy,

$$E_i = E_f + E_k$$

$$\frac{hc}{\lambda_i} = \frac{hc}{\lambda_f} + \frac{1}{2}mv_f^2 \qquad \text{(eq. 1)}$$

From the conservation of momentum,

$$p_i = p_f + p_e$$

$$\frac{h}{\lambda_i} = -\frac{h}{\lambda_f} + mv_f \qquad \text{(eq. 2)}$$

(The negative sign signifies a scatter angle θ equal to 180°.)

Multiplying equation 2 by c and adding the result to equation 1,

$$\frac{2hc}{\lambda_i} = \frac{1}{2}mv_f^2 + cmv_f$$

$$\lambda_i = \frac{2hc}{m\left(\frac{1}{2}v_f^2 + cv_f\right)}$$

$$\lambda_i = \frac{2(6.626 \times 10^{-34}\ \text{J}\cdot\text{s})(3.0 \times 10^{8}\ \text{m/s})}{(9.11 \times 10^{-31}\ \text{kg})\left[\frac{1}{2}(7.12 \times 10^{5}\ \text{m/s})^2 + (3.0 \times 10^{8}\ \text{m/s})(7.12 \times 10^{5}\ \text{m/s})\right]}$$

$$\lambda_i = 2.04 \times 10^{-9}\ \text{m}$$

[12] Physics Book Two, Irwin Publishing, Chapter 12 Problems, #30
[13] Physics Book Two, Irwin Publishing, Chapter 12 Problems, #32
[14] Physics Book Two, Irwin Publishing, Chapter 12 Problems, #34

15. If a photon with an incident wavelength of 18 pm loses 67% of its energy, what is the corresponding change in the photon's wavelength (i.e. Compton shift) as a percentage? [15]

$\lambda_i = 18\text{ pm} = 1.8 \times 10^{-11}\text{ m}$, energy loss is 67 %

The initial energy of the photon can be computed using Planck's equation:

$$E_i = \frac{hc}{\lambda_i}$$

$$E_i = \frac{(6.626 \times 10^{-34}\text{ J}\cdot\text{s})(3.0 \times 10^{8}\text{ m/s})}{1.8 \times 10^{-11}\text{ m}}$$

$$E_i = 1.1 \times 10^{-14}\text{ J}$$

Since 67 % of the energy is lost, the final energy of the photon is:

$$E_f = 0.33E_i$$

$$E_f = 0.33(1.1 \times 10^{-14}\text{ J})$$

$$E_f = 3.64 \times 10^{-15}\text{ J}$$

The final wavelength can be calculated using Planck's equation:

$$\lambda_f = \frac{hc}{E_f}$$

$$\lambda_f = \frac{(6.626 \times 10^{-34}\text{ J}\cdot\text{s})(3.0 \times 10^{8}\text{ m/s})}{3.64 \times 10^{-15}\text{ J}}$$

$$\lambda_f = 5.45 \times 10^{-11}\text{ m}$$

The Compton shift as a percentage is:

$$\frac{\lambda_f}{\lambda_i} = \frac{5.45 \times 10^{-11}\text{ m}}{1.8 \times 10^{-11}\text{ m}} \times 100\,\%$$

$$\frac{\lambda_f}{\lambda_i} = 302\,\%$$

The wavelength of a photon increases by 302 %.

16. A 45-g golf ball is struck and leaves the club at a speed of 50 m/s. What is the de Broglie wavelength associated with this ball? [16]

$m = 45\text{ g} = 0.045\text{ kg}$, $v = 50\text{ m/s}$

Using de Broglie's equation:

$$\lambda = \frac{h}{mv}$$

$$\lambda = \frac{6.626 \times 10^{-34}\text{ J}\cdot\text{s}}{(0.045\text{ kg})(50\text{ m/s})}$$

$$\lambda = 2.9 \times 10^{-34}\text{ m}$$

The wavelength associated with this ball is 2.9×10^{-34} m.

17. What is the de Broglie wavelength of an electron with a kinetic energy of 50 eV? [17]

$E_k = 50\text{ eV} = 8 \times 10^{-18}\text{ J}$,

$m_e = 9.11 \times 10^{-31}\text{ kg}$

a) We shall first compute the velocity using the kinetic energy value:

$$E_k = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2E_k}{m}}$$

$$v = \sqrt{\frac{2(8 \times 10^{-18}\text{ J})}{9.11 \times 10^{-31}\text{ kg}}}$$

$$v = 4.19 \times 10^{6}\text{ m/s}$$

Now λ can be found using de Broglie's equation:

$$\lambda = \frac{h}{mv}$$

$$\lambda = \frac{6.626 \times 10^{-34}\text{ J}\cdot\text{s}}{(9.11 \times 10^{-31}\text{ kg})(4.19 \times 10^{6}\text{ m/s})}$$

$$\lambda = 1.73 \times 10^{-10}\text{ m}$$

[15] Physics Book Two, Irwin Publishing, Chapter 12 Problems, #35
[16] Physics Book Two, Irwin Publishing, Chapter 12 Problems, #36
[17] Physics Book Two, Irwin Publishing, Chapter 12 Problems, #39a

18. In some scattering experiments, the speed of the particles is tuned so that their de Broglie wavelength has a specific value. If a wavelength of 0.117 nm is required, how fast must a neutron be travelling to achieve this wavelength? [18]

$m_n = 1.68 \times 10^{-27}$ kg,

$\lambda = 0.117 \text{ nm} = 1.17 \times 10^{-10}$ m

Using de Broglie's equation:

$$\lambda = \frac{h}{mv}$$

$$v = \frac{h}{m\lambda}$$

$$v = \frac{6.626 \times 10^{-34}\ \text{J}\cdot\text{s}}{(1.68 \times 10^{-27}\ \text{kg})(1.17 \times 10^{-10}\ \text{m})}$$

$v = 3371$ m/s

The velocity of the neutron is 3371 m/s.

19. An electron has a de Broglie wavelength of 7.12×10^{5} m.
    a) What is its momentum?
    b) What is its speed?
    c) What voltage was needed to accelerate it to this speed?[19]

(*a*) We find the momentum from

$$p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34}\ \text{J}\bullet\text{s}}{5.0 \times 10^{-10}\ \text{m}} = \boxed{1.3 \times 10^{-24}\ \text{kg}\bullet\text{m/s.}}$$

(*b*) We find the speed from

$$\lambda = \frac{h}{p} = \frac{h}{mv};$$

$$5.0 \times 10^{-10}\ \text{m} = \frac{6.63 \times 10^{-34}\ \text{J}\bullet\text{s}}{(9.11 \times 10^{-31}\ \text{kg})v}, \text{ which gives } v = \boxed{1.5 \times 10^{6}\ \text{m/s.}}$$

(*c*) With $v < 0.005c$, we can calculate KE classically:

$$\text{KE} = \tfrac{1}{2}mv^2$$

$$= \tfrac{1}{2}(9.11 \times 10^{-31}\ \text{kg})(1.5 \times 10^{6}\ \text{m/s})^2 = 9.7 \times 10^{-19}\ \text{J}, \text{ which converted to electron-volts}$$

equals $\frac{(9.7 \times 10^{-19})}{(1.06 \times 10^{-19}\ \text{J/eV})} = 6.0$ eV. This is the energy gained by an electron as it is accelerated through a potential difference of $\boxed{6.0\,\text{V.}}$

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[18] Physics Book Two, Irwin Publishing, Chapter 12 Problems, #37

[19] Physics 6th Edition, Giancoli, Chapter 27 Problems, #41