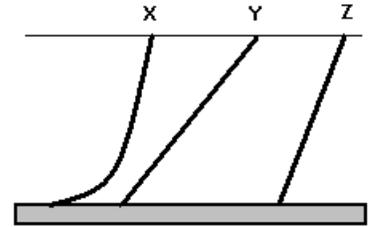


**Conceptual Questions**

1. A child has a choice of three frictionless slides along which to slide to the ground. His speed at the bottom of the slide is the:
- a) fastest for slide X.                      d) the slowest for slide X.  
 b) fastest for slide Z.                      e) fastest for slide Y.  
 c) the same for all three slides.



**Gravitational potential energy is converted to kinetic energy in the same way for all cases**

2. Equal forces were applied to three objects, P, Q, and R, initially at rest on a frictionless surface. If  $m_P < m_Q < m_R$ , which of the objects would have the greatest kinetic energy after traveling the same distance  $d$ ?
- a) P      b) Q      c) R       d) All will have the same kinetic energy.

**Although each will have different speeds, they will all have the same kinetic energy as the work done on each is the same.**

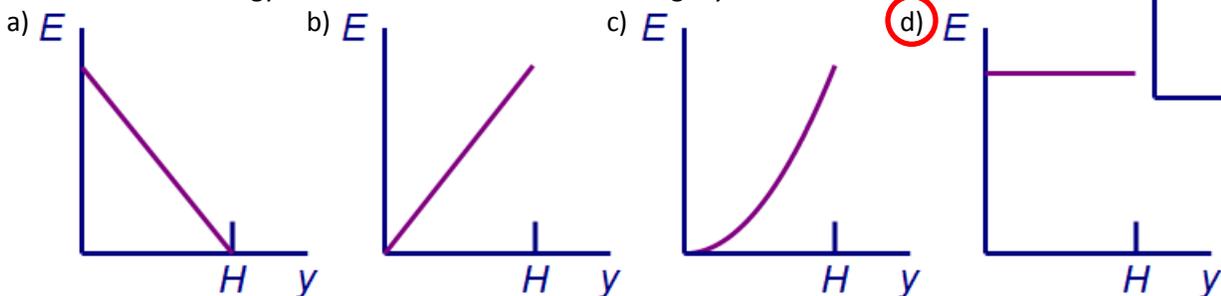
3. A puck starts at rest at the top of a straight, frictionless inclined plane, and slides to the bottom. When it is halfway down the ramp, the puck has
- a) half of the kinetic energy it had at the top.  
 b) half of the potential energy it had at the top.  
 c) half of the speed it had at the bottom.  
 d) taken half of the total time it would spend sliding down the ramp.  
 e) half the total energy it had at the top.

**Halfway down,  $E_g$  is halved since  $h$  is halved.**

4. A skydiver opens her parachute, and falls towards the Earth at a constant speed of 3.6 m/s. As she falls at constant speed, what happens to her gravitational potential energy?
- a) The potential energy becomes kinetic energy of the skydiver.  
 b) The potential energy becomes heat energy due to air resistance.  
 c) Some of the potential energy becomes kinetic energy, and some becomes heat energy.  
 d) All of the potential energy becomes heat energy at the moment she hits the ground.  
 e) Since her speed is constant, her potential energy remains constant as she falls.

**The speed of the skydiver is constant so she/he cannot gain kinetic energy, so all the energy goes into heat (i.e. the motion of air particles).**

5. A ball is held at a height of  $H$  metres above a floor. It is then released and falls to the floor. If air resistance is negligible, which of the graphs shown relates the mechanical energy  $E$  as a function of the vertical height  $y$  of the ball?



**The total energy of the ball is constant ( $W_{nc} = 0$ ).**

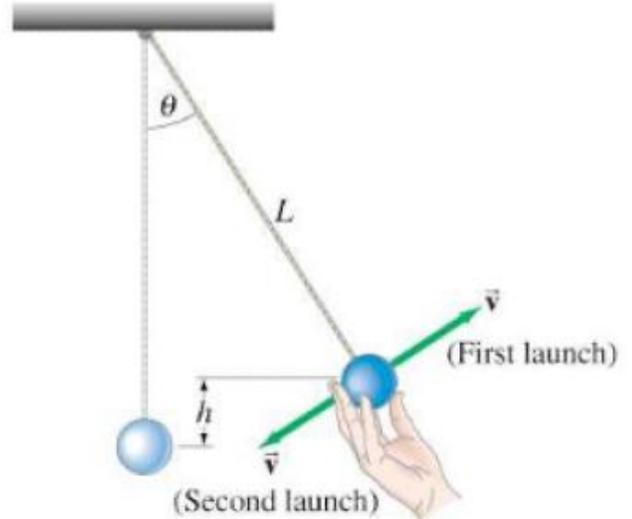
6. A child on a sled (total mass  $m$ ) starts from rest at the top of a hill of height  $h$  and slides down. Does the velocity at the bottom depend on the angle of the hill if a) it is icy and there is no friction, and b) there is friction (deep snow)?

a) No. If there is no energy, all of the  $E_g$  will convert to  $E_k$  regardless of the angle of the hill.

b) Yes. The larger the angle the longer the slope will be. A large angle will increase the force of friction and the longer the slope will increase the distance, both increasing the work done by the surface.

7. A pendulum is launched with an initial speed of 3.0 m/s in two ways: first directly upward along its trajectory, second downwards along its trajectory. Which launch will cause it to swing the largest angle from the equilibrium position? Explain.

Same. The kinetic energy given to the pendulum is independent of which direction the pendulum initially moves, so both cases will convert the same amount of kinetic energy into gravitational potential energy.



### Problems

8. A baseball ( $m=150$  g) travelling at 40 m/s [ $\rightarrow$ ] enters a baseball glove horizontally. If the catcher moves his glove backwards a distance of 12 cm while bringing the ball to rest, calculate the work required to stop the ball.

$$W_{NC} = \Delta E_K + \Delta E_P$$

$$W = (0 J - 0.5mv^2) + 0 J$$

$$W = -0.5(0.15 \text{ kg})(40 \text{ m/s})^2$$

$$W = -120 J$$

9. A ball ( $m=150$  g) that is thrown vertically upward has  $6.0 \times 10^2$  J of initial kinetic energy.

- a) With what speed is it thrown up?

$$E_K = 0.5mv^2$$

$$v = \sqrt{\frac{2E_K}{m}}$$

$$v = \sqrt{\frac{2(600 \text{ J})}{0.15 \text{ kg}}}$$

$$v = 89.44 \text{ m/s}$$

$$v = 89 \text{ m/s}$$

- b) How high will the ball rise? Ignore air friction.

$$W_{NC} = \Delta E_K + \Delta E_P$$

$$0 J = (0 J - 0.5mv^2) + (mgh - 0 J)$$

$$h = \frac{v^2}{2g}$$

$$h = \frac{(89.44 \text{ m/s})^2}{2(9.8 \text{ m/s}^2)}$$

$$h = 410 \text{ m}$$

10. A car travelling at an initial speed of  $V_0$ , slams on the brakes and comes to a halt in distance  $D$ . How far would the same car take (in terms of  $D$ ) coming to rest if it were travelling at  $2V_0$  and the retarding forces were the same as in the first case?

$$W_{NC} = \Delta E_K + \Delta E_P$$

**First case:**

$$FD \cos(\theta) = (0 - 0.5mV_0^2) + 0 J$$

$$F = \frac{mV_0^2}{2D}$$

**Second case:**

$$Fd \cos(\theta) = (0 - 0.5m(2V_0)^2) + 0 J$$

$$\left(\frac{mV_0^2}{2D}\right) d = 0.5m(2V_0)^2$$

$$d = 4D$$

11. An apple ( $m= 100. \text{ g}$ ) drops from a branch of a tree that is  $5.0 \text{ m}$  above the ground.

- a) If air friction can be ignored, with what velocity does the apple hit the ground?

$$W_{NC} = \Delta E_K + \Delta E_P$$

$$0 J = (0.5mv^2 - 0 J) + (0 J - mgh)$$

$$v = \sqrt{2gh}$$

$$v = \sqrt{2(9.8 \text{ m/s}^2)(5.0 \text{ m})}$$

$$v = 9.899 \text{ m/s}$$

$$v = 9.9 \text{ m/s}$$

- b) If the apple actually hits the ground with a velocity of  $6.0 \text{ m/s}$ , what is the average force of friction that acts on the ball as it falls?

$$W_{NC} = \Delta E_K + \Delta E_P$$

$$FD \cos(\theta) = (0.5mv^2 - 0 J) + (0 J - mgh)$$

$$F = \frac{0.5mv^2 - mgh}{-D}$$

$$F = \frac{0.5(0.100 \text{ kg})(6.0 \text{ m/s})^2 - (0.100 \text{ kg})(9.8 \text{ m/s}^2)(5.0 \text{ m})}{-(5.0 \text{ m})}$$

$$F = 0.62 \text{ N}$$

- c) Determine the efficiency of the fall.

$$\text{efficiency} = \frac{E_{OUT}}{E_{IN}} \times 100\%$$

$$\text{efficiency} = \frac{E_{K2}}{E_{g1}} \times 100\%$$

$$\text{efficiency} = \frac{0.5(0.100 \text{ kg})(6.0 \text{ m/s})^2}{(0.100 \text{ kg})(9.8 \text{ m/s}^2)(5.0 \text{ m})} \times 100 \%$$

$$\text{efficiency} = 37 \%$$

12. A child ( $m=50. \text{ kg}$ ) is at the top of a slide so that she is  $3.0 \text{ m}$  vertically above the bottom of the slide. If she gains a speed of  $2.0 \text{ m/s}$  by the time she reaches the bottom of the slide,

- a) calculate the amount of energy lost because of friction.

$$W_{NC} = \Delta E_K + \Delta E_P$$

$$W_{NC} = (0.5mv^2 - 0 J) + (0 J - mgh)$$

$$W_{NC} = 0.5(50 \text{ kg})(2.0 \text{ m/s})^2 - (50 \text{ kg})(9.8 \text{ m/s}^2)(3.0 \text{ m})$$

$$W_{NC} = -1370 \text{ J}$$

**1400 J of energy were lost because of friction**

b) determine the efficiency of the slide.

$$\text{efficiency} = \frac{E_{OUT}}{E_{IN}} \times 100\%$$

$$\text{efficiency} = \frac{E_{K2}}{E_{g1}} \times 100\%$$

$$\text{efficiency} = \frac{0.5(50 \text{ kg})(2.0 \text{ m/s})^2}{(50 \text{ kg})(9.8 \text{ m/s}^2)(3.0 \text{ m})} \times 100 \%$$

$$\text{efficiency} = 6.8 \%$$

13. A block of ice ( $m=4.0 \text{ kg}$ ) starts from rest and slides down a frictionless hill from a height of  $10.0 \text{ m}$ . It strikes a snowman ( $m=7.0 \text{ kg}$ ), initially at rest, at the bottom of the hill and rebounds back up the hill to a height of  $H$  after the collision. If the snowman is moving at  $10.50 \text{ m/s}$  just after the collision, what is the value of  $H$ ?

$$W_{NC} = \Delta E_K + \Delta E_P$$

$$0 \text{ J} = (0.5m_{\text{snowman}}v_{2\text{snowman}}^2 - 0 \text{ J}) + (m_{\text{ice}}gH - m_{\text{ice}}gh_{\text{ice}})$$

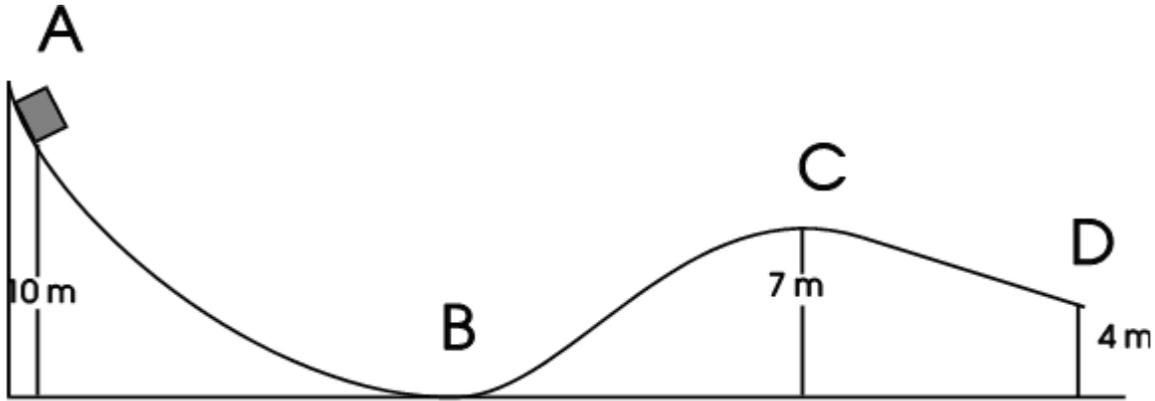
$$H = \frac{m_{\text{ice}}gh_{\text{ice}} - 0.5m_{\text{snowman}}v_{2\text{snowman}}^2}{m_{\text{ice}}g}$$

$$H = \frac{(4.0 \text{ kg})(9.8 \text{ m/s}^2)(10.0 \text{ m}) - 0.5(7.0 \text{ kg})(10.50 \text{ m/s})^2}{(4.0 \text{ kg})(9.8 \text{ m/s}^2)}$$

$$H = 0.156 \text{ m}$$

$$H = 0.16 \text{ m}$$

14.



- a) In the above diagram, calculate the speed of the  $10.0 \text{ kg}$  mass at **B**, **C** and **D**, if it starts from rest at **A**. Assume that no frictional forces exist and that all heights have two significant digits.

**B:**

$$W_{NC} = \Delta E_K + \Delta E_P$$

$$0 \text{ J} = (0.5mv_B^2 - 0 \text{ J}) + (0 \text{ J} - mgh)$$

$$v = \sqrt{2gh}$$

$$v = \sqrt{2(9.8 \text{ m/s}^2)(10 \text{ m})}$$

$$v = 14 \text{ m/s}$$

**D:**

$$W_{NC} = \Delta E_K + \Delta E_P$$

$$0 \text{ J} = (0.5mv_D^2 - 0 \text{ J}) + (mgh_D - mgh_A)$$

$$v = \sqrt{2g(h_A - h_D)}$$

$$v = \sqrt{2(9.8 \text{ m/s}^2)(10 \text{ m} - 4 \text{ m})}$$

$$v = 10.84 \text{ m/s} = 11 \text{ m/s}$$

**C:**

$$W_{NC} = \Delta E_K + \Delta E_P$$

$$0 \text{ J} = (0.5mv_C^2 - 0 \text{ J}) + (mgh_C - mgh_A)$$

$$v = \sqrt{2g(h_A - h_C)}$$

$$v = \sqrt{2(9.8 \text{ m/s}^2)(10 \text{ m} - 7 \text{ m})}$$

$$v = 7.668 \text{ m/s} = 7.7 \text{ m/s}$$

- b) If the speed at **B** is only 1/2 of the value obtained in question 14, how much energy is lost along the trip from **A** to **B**? What would be the efficiency of the slide?

$$W_{NC} = \Delta E_K + \Delta E_P$$

$$W_{NC} = (0.5mv^2 - 0 J) + (0 J - mgh)$$

$$W_{NC} = 0.5(10.0 \text{ kg})(7.0 \text{ m/s})^2 - (10.0 \text{ kg})(9.8 \text{ m/s}^2)(10 \text{ m})$$

$$W_{NC} = -735 \text{ J}$$

**740 J of energy were lost because of friction**

$$\text{efficiency} = \frac{E_{OUT}}{E_{IN}} \times 100\%$$

$$\text{efficiency} = \frac{E_{K2}}{E_{g1}} \times 100\%$$

$$\text{efficiency} = \frac{0.5(10.0 \text{ kg})(7.0 \text{ m/s})^2}{(10.0 \text{ kg})(9.8 \text{ m/s}^2)(10 \text{ m})} \times 100 \%$$

$$\text{efficiency} = 25 \%$$

- c) If the 10.0 kg object does not start from rest but is pushed along the ramp with an initial velocity of 5.0 m/s, what velocity would the object have at B (assume no friction)?

$$W_{NC} = \Delta E_K + \Delta E_P$$

$$0 J = (0.5mv_B^2 - 0.5mv_A^2) + (0 J - mgh_A)$$

$$v = \sqrt{v_A^2 + 2gh_A}$$

$$v = \sqrt{(5.0 \text{ m/s})^2 + 2(9.8 \text{ m/s}^2)(10 \text{ m})}$$

$$v = 14.87 \text{ m/s} = 15 \text{ m/s}$$

- d) If the 10 kg starts from rest and strikes a stationary 12 kg mass at B such that after the collision the 10 kg mass is moving backward at 1.0 m/s, would the 12 kg mass arrive at **C**? Prove it (i.e. show work).

**\*there are multiple ways to prove this. Here is one:**

**Determine the speed of the block at C. If it is less than zero, it will not make it. If it is zero or greater, it will.**

$$W_{NC} = \Delta E_K + \Delta E_P$$

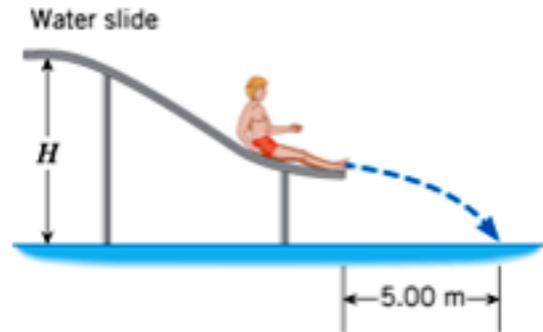
$$0 J = (0.5m_{10kg}v_{10kg B}^2 + 0.5m_{12kg}v_{12kg C}^2 - 0 J) + (m_{12kg}gh_{12kg C} - m_{10kg}gh_{10kg A})$$

$$v_{12kg C} = \sqrt{\frac{2}{m_{12kg}}(m_{10kg}gh_{10kg A} - 0.5m_{10kg}v_{10kg B}^2 - m_{12kg}gh_{12kg C})}$$

$$v_{12kg C} = 4.00 \text{ m/s}$$

**Yes. The speed of the 12 kg object at C would be larger than zero, so it would arrive there.**

15. A water slide is constructed so that swimmers, starting from rest at the top of the slide, leave the end of the slide traveling horizontally. As the drawing shows, one person hits the water 5.00 m from the end of the slide in a time of 0.500 s after leaving the slide. Ignoring friction and air resistance, find the height  $H$  in the drawing.



$$W_{NC} = \Delta E_K + \Delta E_P$$

$$0 J = (0.5mv_{\text{bottom of slide}}^2 - 0 J) + (mgh - mgH) \quad (h = \text{vertical distance from slide to water})$$

$$H = \frac{v_{\text{bottom of slide}}^2}{2g} + h$$

**x direction:**

$$v_{\text{bottom of slide}} = ?$$

$$\vec{a} = 0 \text{ m/s}^2 \text{ [fwd]}$$

$$\Delta \vec{d} = 5.00 \text{ m [fwd]}$$

$$\Delta t = 0.500 \text{ s}$$

$$\Delta \vec{d} = \vec{v}_1 \Delta t + \frac{1}{2} \vec{a} \Delta t^2$$

$$\vec{v}_1 = \frac{\Delta \vec{d}}{\Delta t}$$

$$v_{\text{bottom of slide}} = \frac{5.00 \text{ m [fwd]}}{0.500 \text{ s}}$$

$$v_{\text{bottom of slide}} = 10.0 \text{ m/s}$$

**y direction:**

$$\vec{v}_1 = 0 \text{ m/s [down]}$$

$$\vec{v}_2 = ?$$

$$\vec{a} = 9.8 \text{ m/s}^2 \text{ [down]}$$

$$\Delta t = 0.500 \text{ s}$$

$$\Delta \vec{d} = ?$$

$$\Delta \vec{d} = \vec{v}_1 \Delta t + \frac{1}{2} \vec{a} \Delta t^2$$

$$\Delta \vec{d} = \frac{1}{2} \vec{a} \Delta t^2$$

$$h = \frac{1}{2} (9.8 \text{ m/s}^2) (0.500 \text{ s})^2$$

$$h = 1.225 \text{ m}$$

$$H = \frac{v_{\text{bottom of slide}}^2}{2g} + h$$

$$H = \frac{(10.0 \text{ m/s})^2}{2(9.8 \text{ m/s}^2)} + 1.225 \text{ m}$$

$$H = 6.32704 \text{ m}$$

$$H = 6.33 \text{ m}$$

16. A 2.5 kg block is released from rest at the top of a frictionless 12.2 m high hill. At the bottom of the hill the block encounters a rough, horizontal path whose coefficient of friction is 0.34.

a) How far along this rough path will the block be when moving at 3.8 m/s?

$$W_{NC} = \Delta E_K + \Delta E_P$$

$$F_f d \cos(\theta) = (0.5mv_2^2 - 0J) + (0J - mgh)$$

$$d = -\frac{0.5mv_2^2 - mgh}{\mu F_N}$$

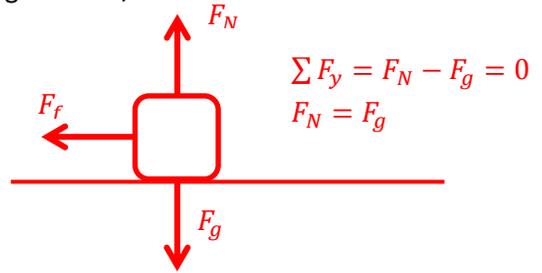
$$d = -\frac{0.5mv_2^2 - mgh}{\mu mg}$$

$$d = \frac{gh - 0.5v_2^2}{\mu g}$$

$$d = \frac{(9.8 \text{ m/s}^2)(12.2 \text{ m}) - 0.5(3.8 \text{ m/s})^2}{(0.34)(9.8 \text{ m/s}^2)}$$

$$d = 33.715 \text{ m}$$

$$d = 34 \text{ m}$$



b) If the block started at the top of the hill with a speed of 4.4 m/s, how far along the rough path would the block travel before coming to rest?

$$W_{NC} = \Delta E_K + \Delta E_P$$

$$F_f d \cos(\theta) = (0J - 0.5mv_1^2) + (0J - mgh)$$

$$d = -\frac{-0.5mv_1^2 - mgh}{\mu F_N}$$

$$d = -\frac{-0.5mv_1^2 - mgh}{\mu mg}$$

$$d = \frac{0.5v_1^2 + gh}{\mu g}$$

$$d = \frac{0.5(4.4 \text{ m/s})^2 + (9.8 \text{ m/s}^2)(12.2 \text{ m})}{(0.34)(9.8 \text{ m/s}^2)}$$

$$d = 38.79 \text{ m}$$

$$d = 39 \text{ m}$$

