Conceptual Questions

1. Imagine standing on the surface of a shrinking planet. If it shrinks to one-tenth its original diameter with no change in mass, on the shrunken surface you'd weigh[1]

A. 1/100 as much. B. 10 times as much. C. 100 times as much.
D. 1000 times as much. E. None of these.

Weight is proportional to $1/r^2$ (or, equivalently, $1/D^2$), so if the diameter is reduced by a factor of 10, your mass would increase by 10^2 or 100 times.

2. A spacecraft on its way from Earth to the Moon is pulled equally by Earth and Moon when it is [2]

A. closer to the Earth's surface. B. closer to the Moon's surface.
C. half way from Earth to Moon . D. At no point, since Earth always pulls more strongly.

Since the moon is less massive than the Earth, the only way to be pulled equally by both is to be closer to the moon to make up for its lack of mass, since its pull increases as you get closer to it (as the Earth's pull decreases).

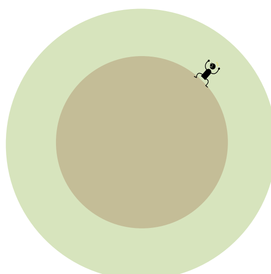
3. If the Sun collapsed to become a black hole, Planet Earth would [3]

A. continue in its present orbit. B. fly off in a tangent path.
C. likely be sucked into the black hole. D. be pulled apart by tidal forces.
E. Both C and D.

The Sun's force of gravitational attraction of the Earth depends solely on its mass and the distance the Earth is from it. Since both these values do not change in the situation described, the force does not change, and so neither does the Earths' orbit.

4. A (theoretical) hole is dug down through the centre of the Earth and out to the other side. What would be your motion if you were to jump into the hole? Explain.

You would speed up towards the centre of the Earth, pass the centre, and then slow down moving away from the centre until stopping at a height equal to your original height, but on the opposite side of the planet. You would then speed up towards the centre again and the process would repeat indefinitely (assuming no frictional forces), oscillating between both ends on the Earth.



However, the rate at which you accelerate (both towards and away from the centre) will not be constant: the assumption that the Earth is a point particle breaks down because you *enter* the Earth. According to [Newton's Shell Theorem], the gravitational field inside a hollow sphere (shell) is zero, so as you enter the Earth, only the remaining spherical mass below your feet pulls you downwards. Since this mass is less than the total mass of the Earth, the acceleration will be less than 9.8 m/s^2 and will decrease as you get closer to the centre. At the centre, the entire Earth acts like a shell so there is no gravitational field and no longer any acceleration. Further explanation can be found here: http://youtu.be/jN-FfJKgis8

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[1] 60 Questions – Basic Physics, Paul G . Hewitt, #15
[2] 60 Questions – Basic Physics, Paul G . Hewitt, #18
[3] 60 Questions – Basic Physics, Paul G . Hewitt, #53

5. The gravitational field strength caused by the Earth on its surface is approximately 9.8 N/kg. At what location would the gravitational field strength caused by the Earth be equal to 0 N/kg? Explain. [4]
   1. **At the centre of the Earth (as described above).**
   2. **Infinitely far away. Due to the inverse square law of gravitational attraction, the field strength asymptotically approaches zero as the distance from the Earth approaches infinity.**

6. The Sun's gravitational pull on the Earth is much larger than the Moon's, yet the moon is mainly responsible for the tides. Explain [*Hint: Consider the difference in gravitational pull from one side of the Earth to the other.*][5]

The tides are caused by the *difference* in the gravitational field strength on opposite sides of the planet. Since the Earth is so far away from the Sun, the difference in distance to the Sun from one side of the Earth compared to the other is negligible, so the difference in gravitational field strength due to the Sun on opposite sides of the Earth is also negligible. However, since the Earth is very close to the moon, the difference in distance to the Moon from one side of the Earth compared to the other is small (about 3% of the total distance), but not negligible. This difference in distance creates a large enough difference in gravitational field strength due to the Moon, which creates the tides.

Problems

7. An object of mass 40.0 kg rests on the surface of a planet with mass 8.2 x10^{22} kg and radius 3.6 x10^{5} m.

a) Calculate the force of gravity acting on the object.

$$F_g = m\frac{Gm}{r^2}$$
$$F_g = (40.0\ kg)\frac{(6.67\times10^{-11}\ N\cdot m^2/kg^2)(8.2\times10^{22}\ kg)}{(3.6\times10^5\ m)^2}$$
$$F_g = 1688.09\ N$$
$$F_g = 1700\ N$$

b) Determine the gravitational field strength "g" at the planet's surface.

$$g = \frac{Gm}{r^2}$$
$$g = \frac{(6.67\times10^{-11}\ N\cdot m^2/kg^2)(8.2\times10^{22}\ kg)}{(3.6\times10^5\ m)^2}$$
$$g = 42.202\ N/kg$$
$$g = 42\ N/kg$$

c) Calculate the force of gravity acting on the object if it is placed at a position 6.4 x10^{5} m above the planet's surface.

$$F_g = m\frac{Gm}{r^2}$$
$$F_g = (40.0\ kg)\frac{(6.67\times10^{-11}\ N\cdot m^2/kg^2)(8.2\times10^{22}\ kg)}{(3.6\times10^5\ m+6.4\times10^5\ m)^2}$$
$$F_g = 218.776\ N$$
$$F_g = 220\ N$$

[4] Almeida, F., Physics Department, Victoria Park C.I.
[5] Physics 6th Edition, Giancoli, Chapter 5 Questions, #14

8. An object of mass 50.0 kg rests at the surface of a planet with a mass of 6.2 x 10^{20} kg and a radius of 3.8 x 10^4 m. What would the object weigh at an altitude equivalent to the planet's radius?

$$F_g = m\frac{Gm}{(2r)^2}$$
$$F_g = (50.0\ kg)\frac{(6.67\times10^{-11}\ N{\cdot}m^2/kg^2)(6.2\times10^{20}\ kg)}{\left(2(3.8\times10^4\ m)\right)^2}$$
$$F_g = 357.981\ N$$
$$F_g = 360\ N$$

9. Your friend explains that astronauts feel weightless while in orbit around the Earth because gravity is really weaker up there in outer space, but is it? Find out yourself by calculating the gravitational field strength of the Earth 380 km above its surface (the orbital altitude of the International Space Station) as a percentage of the gravitational field strength at its surface (9.8 N/kg). [6]

$$g = \frac{Gm}{r^2}$$
$$g = \frac{(6.67\times10^{-11}\ N{\cdot}m^2/kg^2)(5.97\times10^{24}\ kg)}{(6.37\times10^6\ m+380\times10^3\ m)^2}$$
$$g = 8.7340\ N/kg$$

$$g\% = \frac{8.7340\ N/kg}{9.8\ N/kg}\times100\%$$
$$g\% = 89.180\ \%$$
$$g\% = 89\ \%$$

10. The gravitational field strength on the surface of Mars is 3.7 N/kg.

a) What would a person weigh on Mars if this person weighs 637 N on Earth?

$$F_g = m\frac{Gm}{r^2}$$
$$m = \frac{F_g r^2}{Gm}$$
$$m = \frac{(637\ N)\left(6.37\times10^6\ m\right)^2}{(6.67\times10^{-11}\ N{\cdot}m^2/kg^2)(5.97\times10^{24}\ kg)}$$
$$m = 64.911\ kg$$

$$F_g = mg$$
$$F_g = (64.911\ kg)(3.7\ N/kg)$$
$$F_g = 240.171\ N$$
$$F_g = 240\ N$$

b) What is the mass of Mars if its radius is 3.4 x 10^6 m?

$$g = \frac{Gm}{r^2}$$
$$m = \frac{gr^2}{G}$$
$$m = \frac{(3.7\ N/kg)\left(3.4\times10^6\ m\right)^2}{(6.67\times10^{-11}\ N{\cdot}m^2/kg^2)}$$
$$m = 6.41259\times10^{23}\ kg$$
$$m = 6.4\times10^{23}\ kg$$

[6] Physics 6th Edition, Giancoli, Chapter 5 Problems, #38 - modified

11. The gravitational field strength on the surface of some celestial object is 1.6 N/kg and its radius is 1.7 x10^6 m.

a) How much would a 60.0-kg astronaut weigh in orbit at an altitude of 2.0 x10^2 km?

$g = \frac{Gm}{r^2}$

$m = \frac{gr^2}{G}$

$m = \frac{(1.6\ N/kg)(1.7\times10^6\ m)^2}{(6.67\times10^{-11}\ N{\cdot}m^2/kg^2)}$

$m = 6.9325 \times 10^{22}\ kg$

$F_g = m\frac{Gm}{r^2}$

$F_g = (60.0\ kg)\frac{(6.67\times10^{-11}\ N{\cdot}m^2/kg^2)(6.9325\times10^{22}\ kg)}{(1.7\times10^6\ m+2.0\times10^5\ m)^2}$

$F_g = 76.853\ N$

$F_g = 77\ N$

b) If a rock, thrown vertically upward from the Earth's surface, achieves a maximum height of 5.1 m, how high will it reach above the surface of the celestial object if thrown in the same way?

Earth:

$\Delta\vec{d} = 5.1\ m\ [up]$

$\vec{a} = -9.8\ m/s^2\ [up]$

$\vec{v}_2 = 0\ m/s\ [up]$

$\vec{v}_1 = ?$

$v_2^2 = v_1^2 + 2a\Delta d$

$v_1 = (v_2^2 - 2a\Delta d)^{1/2}$

$v_1 = \left((0\ m/s)^2 - 2(-9.8\ m/s^2)\ (5.1\ m)\right)^{1/2}$

$v_1 = 10.00\ m/s$

Celestial Object:

$\Delta\vec{d} = ?$

$\vec{a} = -1.6\ m/s^2\ [up]$

$\vec{v}_2 = 0\ m/s\ [up]$

$\vec{v}_1 = 10.00\ m/s\ [up]$

$v_2^2 = v_1^2 + 2a\Delta d$

$\Delta d = \frac{v_2^2 - v_1^2}{2a}$

$\Delta d = \frac{(0\ m/s)^2-(10.00\ m/s)^2}{2(-1.6\ m/s^2)}$

$\Delta d = 31.25\ m$

$\Delta d = 31\ m$

12. A person stands on a set of bathroom scales (on the Earth) which have been calibrated in Newtons. The scales read 500 N (Assume three significant digits).

a) What would the reading be if the same person stood on the scales on a planet where the gravitational field strength is 14 N/kg?

$F_g = mg$

$m = \frac{F_g}{g}$

$m = \frac{500\ N}{9.8\ N/kg}$

$m = 51.020\ kg$

$F_g = mg$

$F_g = (51.020\ kg)(14\ N/kg)$

$F_g = 714.256\ N$

$F_g = 710\ N$

b) If this planet had a mass of 7.0x10^{24} kg, what would its radius be?

$g = \frac{Gm}{r^2}$

$r = \sqrt{\frac{Gm}{g}}$

$r = \sqrt{\frac{(6.67\times10^{-11}\ N{\cdot}m^2/kg^2)(7.0\times10^{24}\ kg)}{14\ N/kg}}$

$r = 5.7749 \times 10^6\ m$

$r = 5.8 \times 10^6\ m$

13. A 3.0-kg object is dropped from 6.5 m above the surface of Mercury and reaches the ground 1.87 s later.

a) What is the value of the force of gravity exerted by Mercury on the object?

$\Delta\vec{d} = 6.5\ m\ [\downarrow]$

$\vec{a} = ?$

$\vec{v}_1 = 0\ m/s\ [\downarrow]$

$\Delta t = 1.87\ s$

$\Delta\vec{d} = \vec{v}_1\Delta t + \frac{1}{2}\vec{a}\Delta t^2$

$\vec{a} = \frac{2\Delta\vec{d}}{\Delta t^2}$

$\vec{a} = \frac{2(6.5\ m)}{(1.87\ s)^2}$

$\vec{a} = 3.718\ m/s^2$

$F_g = mg$

$F_g = (3.0\ kg)(3.718\ N/kg)$

$F_g = 11.153\ N$

$F_g = 11\ N$

b) If both the Mercury's mass (m = $3.30\text{x}10^{23}$ kg) and radius ($2.44\text{x}10^6$ m) were doubled, how long would it take the object to reach the surface if dropped from the same height?

$$g_{new} = \frac{G(2m_{Mercury})}{(2r_{Mercury})^2} = \frac{2}{4}\frac{Gm_{Mercury}}{r_{Mercury}^2} = \frac{1}{2}g_{Mercury}$$

$$\Delta\vec{d} = \vec{v}_1\Delta t + \frac{1}{2}\vec{a}\Delta t^2$$

$$\Delta t_{new} = \sqrt{\frac{2\Delta d}{g_{new}}} = \sqrt{\frac{2\Delta d}{\frac{1}{2}g_{Mercury}}} = \frac{1}{\sqrt{2}}\sqrt{\frac{2\Delta d}{g_{Mercury}}} = \frac{1}{\sqrt{2}}\Delta t_{Mercury}$$

$$\Delta t_{new} = \frac{1}{\sqrt{2}}(1.87\ s)$$

$$\Delta t_{new} = 1.3223\ s$$

$$\Delta t_{new} = 1.32\ s$$

14. The Earth is the densest planet in the solar system. If it had the density of the least dense planet in the solar system, Saturn (ρ = 0.70 g/cm[3]), how large, compared to its current size (as a percentage), would the Earth need to be for us to still experience the same gravitational field strength on its new surface?[7]

$$g = \frac{Gm}{r^2}, \qquad \rho = \frac{m}{V}, \qquad V_{sphere} = \frac{4}{3}\pi r^3$$

$$g = \frac{Gm}{r^2} = \frac{G\rho V}{r^2} = \frac{G\rho\left(\frac{4}{3}\pi r^3\right)}{r^2} = \frac{4\pi G\rho r}{3}$$

$$r = \frac{3g}{4\pi G\rho}$$

$$r = \frac{3(9.8\ N/\ kg)}{4\pi(6.67\times10^{-11}\ N{\cdot}m^2/kg^2)(0.70\times10^3\ kg/m^3)}$$

$$r = 5.0109\times10^7\ m$$

$$r\% = \frac{5.0109\times10^7\ m}{6.37\times10^6\ m}\times 100\%$$

$$r\% = 786.64\ \%$$

$$r\% = 790\ \%$$

[7] Almeida, F., Physics Department, Victoria Park C.I.