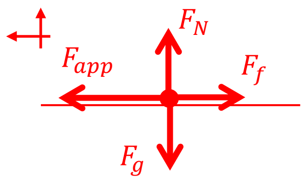
Problems

1. A force of 1.2 N [←] is applied to an object of mass 1.5 kg. It accelerates at 0.50 m/s^2 [←] along a surface. Determine the force of friction that is acting and the coefficient of kinetic friction involved.



$\sum F_x = F_{app} - F_f = ma$

$\sum F_y = F_N - F_g = 0$

From x dir:

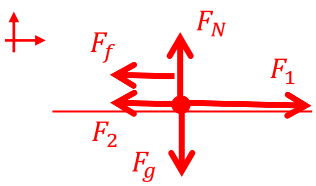
$F_{app} - F_f = ma$
$F_f = F_{app} - ma$
$F_f = 1.2\ N - (1.5\ kg)(0.50\ m/s^2)$
$F_f = 0.45\ N$
$F_f = 0.45\ N\ [\rightarrow]$

From y dir:

$F_N - F_g = 0$
$F_N = F_g$

$F_f = \mu F_N$
$\mu = \frac{F_f}{mg}$
$\mu = \frac{0.45\ N}{(1.5\ kg)(9.8\ m/s^2)}$
$\mu = 0.03$

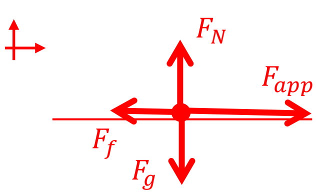
2. Two children pull a toy truck of mass 2.4 kg along a rough horizontal surface. One child pulls with a force of 8.4 N [N] and the other pulls with a force of 3.6 N [S]. The coefficient of friction between the toy and surface is 0.18. What is the acceleration of the toy?



$\sum F_y = F_N - F_g = 0$
$F_N - F_g = 0$
$F_N = F_g$

$\sum F_x = F_1 - F_2 - F_f = ma$
$a = \frac{F_1 - F_2 - \mu mg}{m}$
$a = \frac{8.4\ N - 3.6\ N - (0.18)(2.4\ kg)(9.8\ m/s^2)}{2.4\ kg}$
$a = 0.236\ m/s^2$
$a = 0.24\ m/s^2\ [N]$

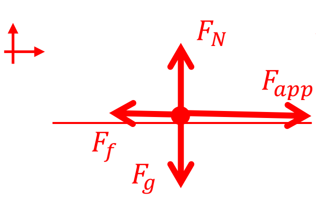
3. What minimum magnitude of force would be required to start a 1.0-kg object sliding along a horizontal surface if the coefficient of static friction is 0.20?



$\sum F_y = F_N - F_g = 0$
$F_N - F_g = 0$
$F_N = F_g$

$\sum F_x = F_{app} - F_f = ma$
$F_{app} - F_f = 0$
$F_{app} = \mu mg$
$F_{app} = (0.20)(1.0\ kg)(9.8\ m/s^2)$
$F_{app} = 1.96\ N$
$F_{app} > 2.0\ N$

4. Show that an applied force of 4.0 N is insufficient to get a stationary 2.0-kg object to move horizontally if the coefficient of starting friction is 0.25.

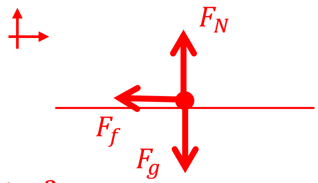


$\sum F_y = F_N - F_g = 0$
$F_N - F_g = 0$
$F_N = F_g$

$\sum F_x = F_{app} - F_f = ma$
$F_{app} - F_f > 0?$
$F_{app} - \mu mg > 0?$
$4.0\ N - (0.25)(2.0\ kg)(9.8\ m/s^2) > 0?$
$-0.9\ N > 0?$

Since the applied force is not large than the force of static friction, their sum is less than 0 N and therefore insufficient to get the object moving.

5. A hockey puck of mass 2.0×10^2 g slides along the ice with a speed of 1.2 m/s when it reaches a rough section where the coefficient of kinetic friction is 0.25. How long (time) will it take the puck to stop sliding? Include a free-body diagram.



$$\sum F_y = F_N - F_g = 0$$
$$F_N - F_g = 0$$
$$F_N = F_g$$

$$\sum F_x = F_f = ma$$
$$\mu mg = ma$$
$$a = \mu g$$

$\Delta t = ?$

$\vec{a} = -\mu g$

$\vec{v}_2 = 0\ m/s$

$\vec{v}_1 = 1.2\ m/s$

$$\vec{v}_2 = \vec{v}_1 + \vec{a}\Delta t$$
$$\Delta t = \frac{\vec{v}_2 - \vec{v}_1}{-\mu g}$$
$$\Delta t = \frac{0\ m/s - 1.2\ m/s}{-(0.25\ kg)(9.8\ m/s^2)}$$
$$\Delta t = 0.4898\ s$$
$$\Delta t = 0.49\ s$$

6. An object of mass 3.8 kg is pushed directly from rest along a horizontal surface, a distance of 1.2 m, and reaches a speed of 2.6 m/s by the end of the push. The frictional force acting is 6.7 N.
   a) Determine the acceleration of the object while being pushed.

$\Delta\vec{d} = 1.2\ m$

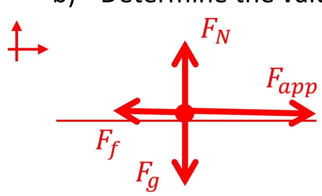
$\vec{a} = ?$

$\vec{v}_2 = 2.6\ m/s$

$\vec{v}_1 = 0\ m/s$

$$v_2^2 = v_1^2 + 2a\Delta d$$
$$a = \frac{v_2^2 - v_1^2}{2\Delta d}$$
$$a = \frac{(2.6\ m/s)^2 - (0\ m/s)^2}{2(1.2\ m)}$$
$$a = 2.81667\ m/s$$
$$a = 2.8\ m/s^2\ [\rightarrow]$$

   b) Determine the value of the applied force that is acting.

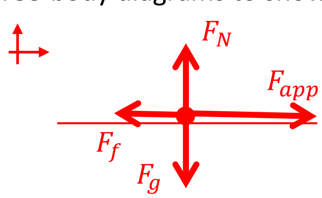


$$\sum F_y = F_N - F_g = 0$$
$$F_N - F_g = 0$$
$$F_N = F_g$$

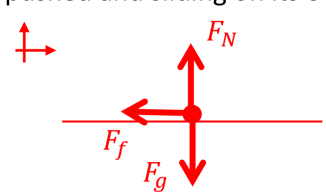
$$\sum F_x = F_{app} - F_f = ma$$
$$F_{app} = ma + F_f$$
$$F_{app} = (3.8\ kg)(2.81667\ m/s^2) + 6.7\ N$$
$$F_{app} = 17.4033\ N$$
$$F_{app} = 17\ N\ [\rightarrow]$$

7. A stationary box of mass 4.2 kg is given a push of 8.2 N [S] along a surface where the frictional force acting is 5.8 N [N]. The push lasts for 3.6 s and then the box is allowed to slide on its own until it comes to rest.
   a) Draw free-body diagrams to show the box being pushed and sliding on its own.

Pushed:



Sliding:



   b) Determine the acceleration of the box as it is being pushed.

$$\sum F_y = F_N - F_g = 0$$
$$F_N - F_g = 0$$
$$F_N = F_g$$

$$\sum F_x = F_{app} - F_f = ma$$
$$a = \frac{F_{app} - F_f}{m}$$
$$a = \frac{8.2\ N - 5.8\ N}{4.2\ kg}$$

$$a = 0.5714\ m/s^2$$
$$a = 0.57\ m/s^2\ [S]$$

c) Calculate the speed of the box just as the push ceases.

$\Delta t = 0.823\ s$ $\qquad$ $\vec{v}_2 = \vec{v}_1 + \vec{a}\Delta t$

$\vec{a} = 0.57\ m/s^2$ $\qquad$ $\vec{v}_2 = 0\ m/s + (0.5714\ m/s^2)(3.6\ s)$

$\vec{v}_2 = ?$ $\qquad$ $\vec{v}_2 = 2.0571\ m/s$

$\vec{v}_1 = 0\ m/s$ $\qquad$ $\vec{v}_2 = 2.1\ m/s\ [S]$

d) Determine the acceleration of the box as it is sliding on its own and the time it takes to come to rest.

$\sum F_y = F_N - F_g = 0$

$F_N - F_g = 0$

$F_N = F_g$

$\sum F_x = -F_f = -ma$

$a = \frac{F_f}{m}$

$a = \frac{5.8\ N}{4.2\ kg}$

$a = 1.3810\ m/s^2$

$a = 1.4\ m/s^2\ [N]$

$\Delta t = ?$

$\vec{a} = -1.3810\ m/s^2$

$\vec{v}_2 = 0\ m/s$

$\vec{v}_1 = 2.1\ m/s$

$\vec{v}_2 = \vec{v}_1 + \vec{a}\Delta t$

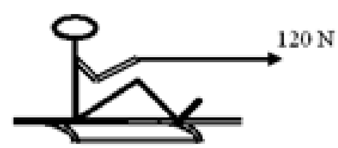
$\Delta t = \frac{\vec{v}_2 - \vec{v}_1}{a}$

$\Delta t = \frac{0\ m/s - 2.1 m/s}{-1.3810\ m/s^2}$

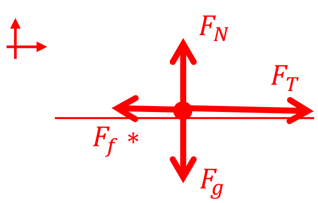
$\Delta t = 1.5206\ s$

$\Delta t = 1.5\ s$

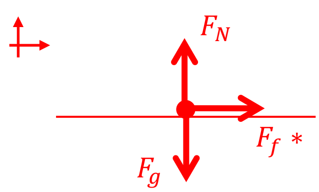
8. A child with mass 30 kg was sitting on a sled with mass 10 kg. The friction between the sled and the snow was negligible. If a force of 120 N was applied to the child, as shown, what was the minimum coefficient of static friction required between the child and the sled to keep the child from slipping off?



Child:



Sled:



$*$ *between child and sled*

$\sum F_y = F_N - F_g = 0$

$\sum F_x = F_T - F_f = ma$

$\sum F_y = F_N - F_g = 0$

$\sum F_x = F_f = ma$

In order for the child not to slip off of the sled, the child and the sled must have the same acceleration:

$a_C = \frac{F_T - F_f}{m_C}$ $\qquad$ $a_S = \frac{F_f}{m_S}$

$$\frac{F_T - F_f}{m_C} = \frac{F_f}{m_S}$$

$$F_T = \frac{m_C}{m_S}F_f + F_f$$

$$F_f = \frac{F_T}{1 + \frac{m_C}{m_S}}$$

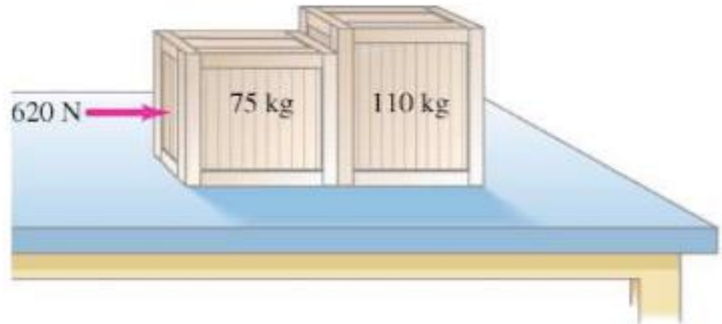
$$\mu = \frac{F_T}{m_C g\left(1 + \frac{m_C}{m_S}\right)}$$

$$\mu = \frac{120\ N}{(30\ kg)(9.8\ m/s^2)\left(1 + \frac{30\ kg}{10\ kg}\right)}$$

$$\mu = 0.10204$$

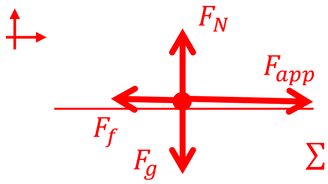
$$\mu = 0.10$$

9. Two crates, of mass 75 kg and 110 kg, are in contact and at rest on a horizontal surface. A 620-N force is exerted on the 75-kg crate. The coefficient of kinetic friction, for both crates, is 0.15.



a) Calculate the acceleration of the system

Considering both crates as one:

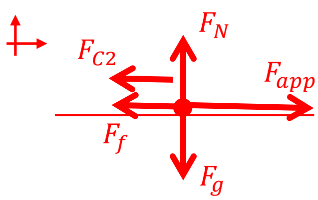


$$\sum F_y = F_N - F_g = 0$$
$$F_N - F_g = 0$$
$$F_N = F_g$$

$$\sum F_x = F_{app} - F_f = ma$$
$$a = \frac{F_{app} - \mu mg}{m}$$
$$a = \frac{620\ N - (0.15)(75\ kg + 110\ kg)(9.8\ m/s^2)}{(75\ kg + 110\ kg)}$$
$$a = 1.8813\ m/s^2$$
$$a = 1.9\ m/s^2\ [\rightarrow]$$

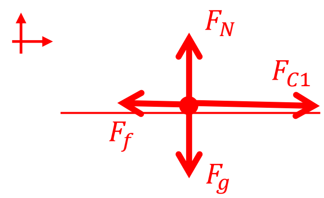
b) Calculate the force that each crate exerts on the other

Crate1:



$$\sum F_y = F_N - F_g = 0$$
$$\sum F_x = F_{app} - F_{C2} - F_f = ma$$

Crate2:



$$\sum F_y = F_N - F_g = 0$$
$$\sum F_x = F_{C1} - F_f = ma$$

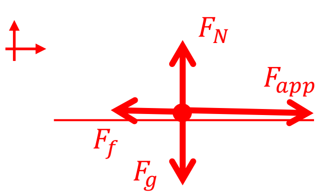
Selecting crate2:

$$F_{C1} - F_f = ma$$
$$F_{C1} = ma + \mu mg$$
$$F_{C1} = m(a + \mu g)$$

$$F_{C1} = (110\ kg)\left(1.8813\ m/s^2 + (0.15)(9.8\ m/s^2)\right)$$
$$F_{C1} = 368.643\ N$$
$$F_{C1} = 370\ N$$

c) Repeat a) and b) with the crates reversed

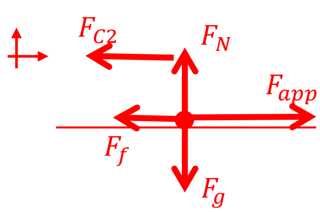
Considering both crates as one:



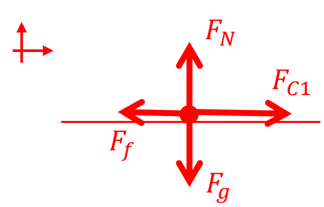
$$\sum F_y = F_N - F_g = 0$$
$$F_N - F_g = 0$$
$$F_N = F_g$$

$$\sum F_x = F_{app} - F_f = ma$$
$$a = \frac{F_{app} - \mu mg}{m}$$
$$a = \frac{620\ N - (0.15)(75\ kg + 110\ kg)(9.8\ m/s^2)}{(75\ kg + 110\ kg)}$$
$$a = 1.8813\ m/s^2$$
$$a = 1.9\ m/s^2\ [\rightarrow]$$

Crate1:



Crate2:



$\sum F_y = F_N - F_g = 0$
$\sum F_x = F_{app} - F_{C2} - F_f = ma$

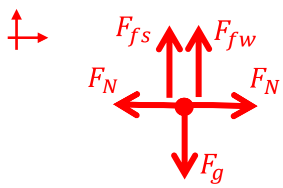
$\sum F_y = F_N - F_g = 0$
$\sum F_x = F_{C1} - F_f = ma$

Selecting crate2:

$F_{C1} - F_f = ma$
$F_{C1} = ma + \mu mg$
$F_{C1} = m(a + \mu g)$

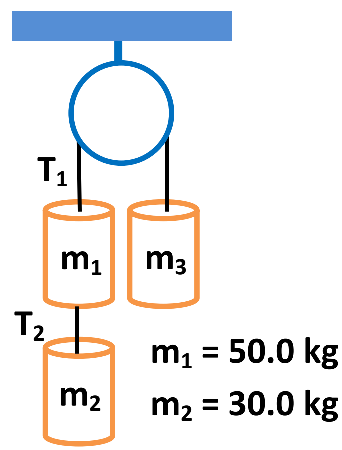
$F_{C1} = (75\ kg)(1.8813\ m/s^2\ + (0.15)(9.8\ m/s^2))$
$F_{C1} = 251.3475\ N$
$F_{C1} = 250\ N$

10. A 75-kg climber is supported in the "chimney" by the friction forces exerted on his shoes and back. The static coefficients of friction between his shoes and the wall, and between his back and the wall, are 0.80 and 0.60 respectively. What is the minimum normal force he must exert? Assume the walls are vertical and that friction forces are both at a maximum. Ignore his grip on the rope.



$\sum F_y = F_{fs} + F_{fw} - F_g = 0$
$\sum F_x = F_N - F_N = 0$

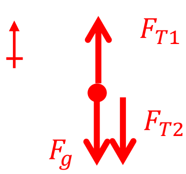
$\mu_s F_N + \mu_w F_N - mg = 0$
$F_N(\mu_s + \mu_w) = mg$
$F_N = \frac{mg}{(\mu_s + \mu_w)}$
$F_N = \frac{(75\ kg)(9.8\ m/s^2)}{(0.80+060)}$
$F_N = 525\ N$
$F_N = 520\ N$



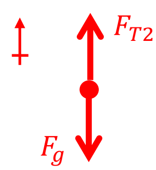
11.

a) If $m_3 = 1.00{\times}10^2$ kg, then calculate the acceleration of the system and the tension in each string.

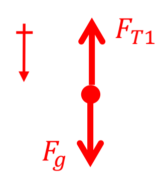
mass 1:



mass 2:



mass 3:



$\sum F_y = F_{T1} - F_{T2} - F_g = ma$

$\sum F_y = F_{T2} - F_g = ma$
$F_{T2} = m_2 a + m_2 g$

$\sum F_y = F_g - F_{T1} = ma$
$F_{T1} = m_3 g - m_3 a$

$F_{T1} - F_{T2} - F_g = ma$
$(m_3 g - m_3 a) - (m_2 a + m_2 g) - m_1 g = m_1 a$
$a(m_1 + m_2 + m_3) = g(m_3 - m_2 - m_1)$
$a = \frac{g(m_3 - m_2 - m_1)}{(m_1 + m_2 + m_3)}$
$a = \frac{(9.8\ m/s^2)(100\ kg - 30.0\ kg - 50.0\ kg)}{(50.0\ kg + 30.0\ kg + 100\ kg)}$
$a = 1.0889\ m/s^2$
$a = 1.09\ m/s^2$

$$F_{T1} = m_3 g - m_3 a$$
$$F_{T1} = (100\ kg)(9.8\ m/s^2 - 1.0889\ m/s^2)$$
$$F_{T1} = 871.11\ N$$
$$F_{T1} = 871\ N$$

$$F_{T2} = m_2 a + m_2 g$$
$$F_{T2} = (30.0\ kg)(1.0889\ m/s^2 + 9.8\ m/s^2)$$
$$F_{T2} = 326.667\ N$$
$$F_{T2} = 327\ N$$

b) If T_2 has a breaking strength of 450 N, then what value of m_3 would be the maximum?

mass 1:

$$\sum F_y = F_{T1} - F_{T2} - F_g = ma$$

mass 2:

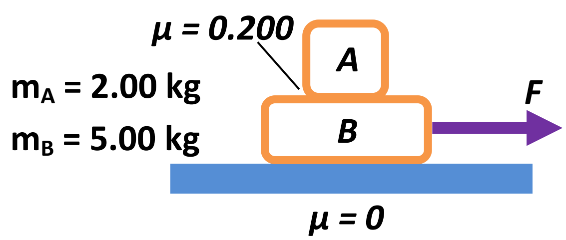
$$\sum F_y = F_{T2} - F_g = ma$$
$$a = \frac{F_{T2} - m_2 g}{m_2}$$

mass 3:

$$\sum F_y = F_g - F_{T1} = ma$$
$$F_{T1} = m_3 g - m_3 a$$

$$F_{T1} - F_{T2} - F_g = ma$$
$$\left(m_3 g - m_3\left(\frac{F_{T2} - m_2 g}{m_2}\right)\right) - F_{T2} - m_1 g = m_1\left(\frac{F_{T2} - m_2 g}{m_2}\right)$$
$$m_3\left(g - \left(\frac{F_{T2} - m_2 g}{m_2}\right)\right) = m_1\left(\frac{F_{T2} - m_2 g}{m_2}\right) + F_{T2} + m_1 g$$
$$m_3 = \frac{m_1\left(\frac{F_{T2} - m_2 g}{m_2}\right) + F_{T2} + m_1 g}{\left(g - \left(\frac{F_{T2} - m_2 g}{m_2}\right)\right)}$$
$$m_3 = \frac{(50.0\ kg)\left(\frac{450\ N - (30.0\ kg)(9.8\ m/s^2)}{30.0\ kg}\right) + 450\ N + (50.0\ kg)(9.8\ m/s^2)}{\left((9.8\ m/s^2) - \left(\frac{450\ N - (30.0\ kg)(9.8\ m/s^2)}{30.0\ kg}\right)\right)}$$
$$m_3 = 260.870\ kg$$
$$m_3 = 261\ kg$$

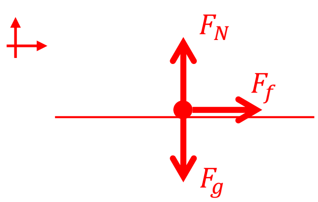
12.



a) Identify the type of force that accelerates block ***A***.
force of friction between block *A* and block *B*

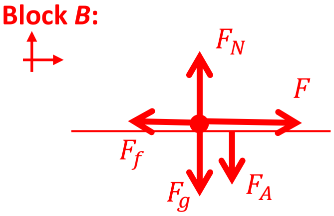
b) Calculate the force required to accelerate block ***B*** to the right at a rate of 3.0 m/s². Calculate the acceleration of block ***A***.

Block *A*:



$$\sum F_y = F_N - F_g = 0$$
$$\sum F_x = F_f = ma$$

Block *B*:



$$\sum F_y = F_N - F_g - F_A = 0$$
$$\sum F_x = F - F_f = ma$$

from block *A*:

$$F_N - F_g = 0$$
$$F_N = m_A g$$

from block *B*:

$$F - F_f = ma$$
$$F = m_B a + \mu F_N$$
$$F = m_B a + \mu m_A g$$
$$F = (5.00\ kg)(3.0\ m/s^2) + (0.200)(2.00\ kg)(9.8\ m/s^2)$$
$$F = 18.92\ N$$
$$F = 18.9\ N$$

from block *A*:

$F_N - F_g = 0$

$F_N = m_A g$

$F_f = ma$

$a = \frac{\mu F_N}{m_A}$

$a = \frac{\mu m_A g}{m_A}$

$a = \mu g$

$a = (0.200)(9.8\ m/s^2)$

$a = 1.96\ m/s^2$

c) Find the minimum coefficient of friction between the two blocks so that block ***A*** doesn't move relative to block ***B*** as block ***B*** accelerates at 3.0 m/s^2.

from block *A*:

$F_N - F_g = 0$

$F_N = m_A g$

$F_f = ma$

$\mu = \frac{m_A a}{F_N}$

$\mu = \frac{m_A a}{\mu m_A g}$

$\mu = \frac{a}{g}$

$F = \frac{3.0\ m/s^2}{9.8\ m/s^2}(9.8\ m/s^2)$

$F = 0.30612$

$F = 0.306$

d) Find the maximum force ***F*** so that block ***A*** doesn't slide with respect to block ***B*** (***μ*** = 0.200).

from block *A*:

$F_f = ma$

$a = \frac{F_f}{m_A}$

from block *B*:

$F - F_f = ma$

$a = \frac{F - F_f}{m_B}$

If block *A* doesn't slide, both objects will have the same acceleration:

$\frac{F_f}{m_A} = \frac{F - F_f}{m_B}$

$m_A F = m_B F_f + m_A F_f$

$F = \frac{\mu m_A g (m_B + m_A)}{m_A}$

$F = \frac{(0.200)(2.00\ kg)(9.8\ m/s^2)(5.00\ kg + 2.00\ kg)}{2.00\ kg}$

$F = 13.72\ N$

$F = 13.7\ N$