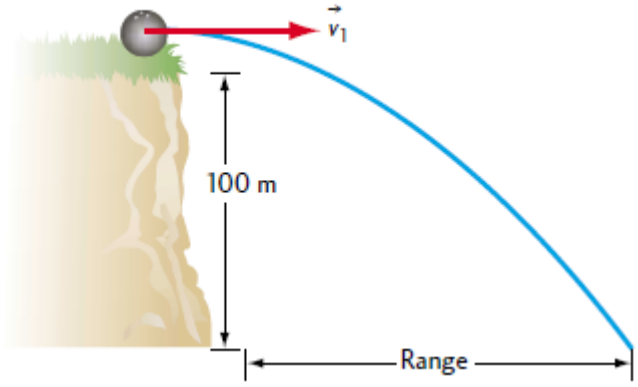
2-D Horizontal Projectile Problems



1. A bowling ball is rolled off the top of a cliff with an initial horizontal velocity of 6.0 m/s [E]. If the cliff is 100. m above the ground, determine

a) the ball's time of flight

y direction:

$\vec{v}_1 = 0\ m/s\ [down]$

$\vec{v}_2 = ?$

$\vec{a} = 9.8\ m/s^2\ [down]$

$\Delta\vec{d} = 1.0 \times 10^2\ m\ [down]$

$\Delta t = ?$

$\Delta\vec{d} = \vec{v}_1\Delta t + \frac{1}{2}\vec{a}\Delta t^2$

$\Delta t = \sqrt{\frac{2\Delta\vec{d}}{\vec{a}}}$

$\Delta t = \sqrt{\frac{2(1.0\times10^2\ m)}{9.8\ m/s^2}}$

$\Delta t = -4.518\ s,\ 4.518\ s$

$\Delta t = 4.5\ s$

b) the ball's range

x direction:

$\vec{v}_1 = 6.0\ m/s\ [E]$

$\vec{v}_2 = 6.0\ m/s\ [E]$

$\vec{a} = 0\ m/s^2\ [E]$

$\Delta\vec{d} = ?$

$\Delta t = 4.5\ s$

$\Delta\vec{d} = \vec{v}_1\Delta t + \frac{1}{2}\vec{a}\Delta t^2$

$\Delta\vec{d} = \vec{v}_1\Delta t$

$\Delta\vec{d} = (6.0\ m/s)(4.5\ s)$

$\Delta\vec{d} = 27\ m\ [E]$

2. A helicopter flying horizontally at a velocity of 25 m/s drops a mailbag from a height of 15m to a letter carrier waiting on the ground below.

a) How long will the bag take to fall to the ground?

y direction:

$\vec{v}_1 = 0\ m/s\ [down]$

$\vec{v}_2 = ?$

$\vec{a} = 9.8\ m/s^2\ [down]$

$\Delta\vec{d} = \vec{v}_1\Delta t + \frac{1}{2}\vec{a}\Delta t^2$

$\Delta t = \sqrt{\frac{2\Delta\vec{d}}{\vec{a}}}$

$\Delta t = \sqrt{\frac{2(15\ m)}{9.8\ m/s^2}}$

$\Delta\vec{d} = 15\ m\ [down]$
$\Delta t = ?$

$\Delta t = -1.749\ s,\ 1.749\ s$
$\Delta t = 1.7\ s$

c) How far in advance of the letter carrier must the bag be released so that it lands at her feet?

x direction:

$\vec{v}_1 = 25\ m/s\ [E]$
$\vec{v}_2 = 25\ m/s\ [E]$
$\vec{a} = 0\ m/s^2\ [E]$
$\Delta\vec{d} = ?$
$\Delta t = 1.7\ s$

$\Delta\vec{d} = \vec{v}_1\Delta t + \frac{1}{2}\vec{a}\Delta t^2$
$\Delta\vec{d} = \vec{v}_1\Delta t$
$\Delta\vec{d} = (25\ m/s)(1.8\ s)$
$\Delta\vec{d} = 42\ m\ [E]$

Must release mailbag 42 m before carrier.

c) What will be the mailbag's final velocity?

x direction:

$\vec{v}_{2x} = 25\ m/s\ [E]$

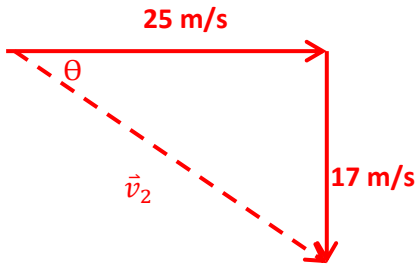
y direction:

$\vec{v}_1 = 0\ m/s\ [down]$
$\vec{a} = 9.8\ m/s^2\ [down]$
$\Delta\vec{d} = 15\ m\ [down]$
$\vec{v}_2 = ?$

$v_2^2 = v_1^2 + 2a\Delta d$
$v_2 = \sqrt{2a\Delta d}$
$v_2 = \sqrt{2(9.8\ m/s^2)(15\ m)}$
$v_2 = 17.15\ m/s$
$\vec{v}_{2y} = 17\ m/s\ [down]$

$\Delta d = \sqrt{(25\ m)^2 + (17\ m)^2}$
$\Delta d = 30.23\ m/s$
$\Delta d = 3.0 \times 10^1\ m/s$

$\tan(\theta) = \frac{17\ m/s}{25\ m/s}$
$\theta = \tan^{-1}\left(\frac{17\ m/s}{25\ m/s}\right)$
$\theta = 34°$



$\Delta\vec{d} = 3.0 \times 10^1\ m/s[E34°down]$

3. Ms. Swanson throws a tomato horizontally out of an open window with a velocity of 3.0 m/s. If the window is 10. m above the ground, how far away from the building must Ms. Reichling stand to catch the ball at ground level?

x direction:

$\vec{v}_1 = 3.0\ m/s\ [fwd]$
$\vec{v}_2 = 3.0\ m/s\ [fwd]$
$\vec{a} = 0\ m/s^2\ [fwd]$
$\Delta\vec{d} = ?$
$\Delta t = ?$

y direction:

$\vec{v}_1 = 0\ m/s\ [down]$
$\vec{v}_2 = ?$
$\vec{a} = 9.8\ m/s^2\ [down]$
$\Delta\vec{d} = 1.0 \times 10^1\ m\ [down]$
$\Delta t = ?$

From y direction:

$\Delta\vec{d} = \vec{v}_1\Delta t + \frac{1}{2}\vec{a}\Delta t^2$
$\Delta t = \sqrt{\frac{2\Delta\vec{d}}{\vec{a}}}$
$\Delta t = \sqrt{\frac{2(10\ m)}{9.8\ m/s^2}}$
$\Delta t = -1.429\ s,\ 1.429\ s$
$\Delta t = 1.4\ s$

From x direction:

$\Delta\vec{d} = \vec{v}_1\Delta t + \frac{1}{2}\vec{a}\Delta t^2$
$\Delta\vec{d} = \vec{v}_1\Delta t$
$\Delta\vec{d} = (3.0\ m/s)(1.4\ s)$
$\Delta\vec{d} = 4.2\ m\ [fwd]$

She should stand 4.2 m in front of building.

4. A rock thrown horizontally from the top of a water tower lands 20.0 m from the base of the tower. If the rock was initially thrown at a velocity of 10.0 m/s,

a) how high is the water tower?

x direction:

$\vec{v}_1 = 10.0\ m/s\ [fwd]$
$\vec{v}_2 = 10.0\ m/s\ [fwd]$
$\vec{a} = 0\ m/s^2\ [fwd]$
$\Delta\vec{d} = 20.0\ m\ [fwd]$
$\Delta t = ?$

y direction:

$\vec{v}_1 = 0\ m/s\ [down]$
$\vec{v}_2 = ?$
$\vec{a} = 9.8\ m/s^2\ [down]$
$\Delta\vec{d} = ?$
$\Delta t = ?$

From x direction:

$\Delta\vec{d} = \vec{v}_1\Delta t + \frac{1}{2}\vec{a}\Delta t^2$
$\Delta t = \frac{\Delta\vec{d}}{\vec{v}_1}$

From y direction:

$\Delta\vec{d} = \vec{v}_1\Delta t + \frac{1}{2}\vec{a}\Delta t^2$
$\Delta\vec{d} = \frac{1}{2}\vec{a}\Delta t^2$

$$\Delta t = \frac{20.0\ m}{10.0\ m/s}$$

$$\Delta t = 2.00\ s$$

$$\Delta \vec{d} = \tfrac{1}{2}(9.8\ m/s^2)(2.00\ s)^2$$

$$\Delta \vec{d} = 19.6\ m\ [down]$$

b) with what final velocity will the rock strike the ground?

From y direction:

$$v_2^2 = v_1^2 + 2a\Delta d$$

$$v_2 = \sqrt{2a\Delta d}$$

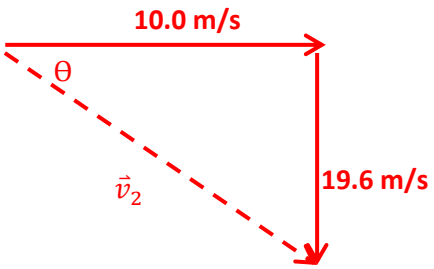
$$v_2 = \sqrt{2(9.8\ m/s^2)(19.6\ m)}$$

$$v_2 = 19.6\ m/s$$

Combining directions:

$$\vec{v}_{2x} = 10.0\ m/s\ [fwd]$$

$$\vec{v}_{2y} = 19.6\ m/s\ [down]$$



$$v_2 = \sqrt{(v_{2x})^2 + (v_{2y})^2}$$

$$v_2 = \sqrt{(10.0\ m/s)^2 + (19.6\ m/s)^2}$$

$$v_2 = 22.00\ m/s$$

$$v_2 = 22.0\ m/s$$

$$\tan(\theta) = \frac{19.6\ m/s}{10.0\ m/s}$$

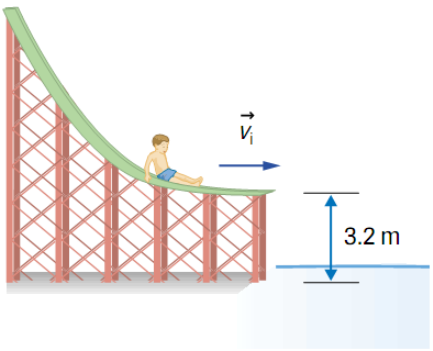
$$\theta = \tan^{-1}\left(\frac{(19.6\ m/s)}{10.0\ m/s}\right)$$

$$\theta = 63°$$

$$\vec{v}_2 = 22\ m/s\ [fwd 63° down]$$

5. A child travels down a waterslide, leaving it with a velocity of 4.2 m/s horizontally, as in the picture below. The child then experiences projectile motion, landing in a swimming pool 3.2m below the slide.

a) For how long is the child airborne?



y direction:

$\vec{v}_1 = 0\ m/s\ [down]$

$\vec{v}_2 = ?$

$\vec{a} = 9.8\ m/s^2\ [down]$

$\Delta\vec{d} = 15\ m\ [down]$

$\Delta t = ?$

$\Delta\vec{d} = \vec{v}_1\Delta t + \frac{1}{2}\vec{a}\Delta t^2$

$\Delta t = \sqrt{\frac{2\Delta\vec{d}}{\vec{a}}}$

$\Delta t = \sqrt{\frac{2(3.2\ m)}{9.8\ m/s^2}}$

$\Delta t = -0.808\ s,\ 0.808\ s$

$\Delta t = 0.81\ s$

b) Determine the child's horizontal displacement travelling through in the air.

x direction:

$\vec{v}_1 = 4.2\ m/s\ [fwd]$

$\vec{v}_2 = 4.2\ m/s\ [fwd]$

$\vec{a} = 0\ m/s^2\ [fwd]$

$\Delta\vec{d} = ?$

$\Delta t = 0.81\ s$

$\Delta\vec{d} = \vec{v}_1\Delta t + \frac{1}{2}\vec{a}\Delta t^2$

$\Delta\vec{d} = \vec{v}_1\Delta t$

$\Delta\vec{d} = (4.2\ m/s)(0.81\ s)$

$\Delta\vec{d} = 3.4\ m\ [fwd]$

6. A helicopter, travelling horizontally, is 82m above the ground. The pilot prepares to release a relief package intended to land on the ground 96 m horizontally ahead. Air resistance is negligible. The pilot does not throw the package, but lets it drop.

a) What is the initial velocity of the package relative to the ground?

x direction:

$\vec{v}_1 = ?$

$\vec{v}_2 = ?$

$\vec{a} = 0\ m/s^2\ [fwd]$

$\Delta\vec{d} = 96\ m\ [fwd]$

$\Delta t = ?$

y direction:

$\vec{v}_1 = 0\ m/s\ [down]$

$\vec{v}_2 = ?$

$\vec{a} = 9.8\ m/s^2\ [down]$

$\Delta\vec{d} = 82\ m\ [down]$

$\Delta t = ?$

From y direction:

$\Delta\vec{d} = \vec{v}_1\Delta t + \frac{1}{2}\vec{a}\Delta t^2$

$\Delta t = \sqrt{\frac{2\Delta\vec{d}}{\vec{a}}}$

From x direction:

$\Delta\vec{d} = \vec{v}_1\Delta t + \frac{1}{2}\vec{a}\Delta t^2$

$\vec{v}_1 = \frac{\Delta\vec{d}}{\Delta t}$

$$\Delta t = \sqrt{\frac{2(82\ m)}{9.8\ m/s^2}}$$
$$\Delta t = -4.091\ s, 4.091\ s$$
$$\Delta t = 4.09\ s$$

$$\vec{v}_1 = \frac{96}{4.09}$$
$$\vec{v}_1 = 23.47\ m/s$$
$$\vec{v}_1 = 23\ m/s\ [fwd]$$

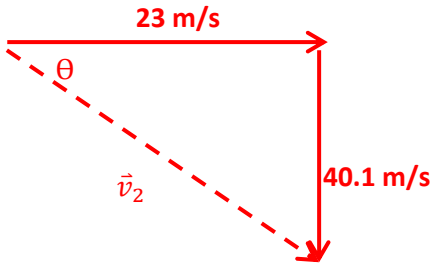
b) What is the final velocity of the package relative to the ground?

From y direction:

$$v_2^2 = v_1^2 + 2a\Delta d$$
$$v_2 = \sqrt{2a\Delta d}$$
$$v_2 = \sqrt{2(9.8\ m/s^2)(82\ m)}$$
$$v_2 = 40.09\ m/s$$

Combining directions:

$$\vec{v}_{2x} = 23\ m/s\ [fwd]$$
$$\vec{v}_{2y} = 40.1\ m/s\ [down]$$



$$v_2 = \sqrt{(v_{2x})^2 + (v_{2y})^2}$$
$$v_2 = \sqrt{(23\ m/s)^2 + (40.1\ m/s)^2}$$
$$v_2 = 46.23\ m/s$$
$$v_2 = 46\ m/s$$

$$\tan(\theta) = \frac{40.1\ m/s}{23\ m/s}$$
$$\theta = \tan^{-1}\left(\frac{40.1\ m/s}{23\ m/s}\right)$$
$$\theta = 60°$$

$$\vec{v}_2 = 46\ m/s\ [fwd60°down]$$

7. A child throws a snowball with a horizontal velocity of 18 m/s directly toward a tree, from a distance of 9.0 m and a height above the ground of 1.5 m.

a) After what time interval does the snowball hit the tree?

x direction:

$$\vec{v}_1 = 18\ m/s\ [fwd]$$
$$\vec{v}_2 = 18\ m/s\ [fwd]$$
$$\vec{a} = 0\ m/s^2\ [fwd]$$

$$\Delta\vec{d} = \vec{v}_1\Delta t + \tfrac{1}{2}\vec{a}\Delta t^2$$
$$\Delta t = \frac{\Delta\vec{d}}{\vec{v}_1}$$
$$\Delta t = \frac{9.0\ m}{18\ m/s}$$

$\Delta\vec{d} = 9.0\ m\ [fwd]$ $\qquad$ $\Delta t = 0.50\ s$

$\Delta t = ?$

b) At what height above the ground will the snow-ball hit the tree?

y direction:

$\vec{v}_1 = 0\ m/s\ [down]$ $\qquad$ $\Delta\vec{d} = \vec{v}_1\Delta t + \frac{1}{2}\vec{a}\Delta t^2$

$\vec{v}_2 = ?$ $\qquad$ $\Delta\vec{d} = \frac{1}{2}\vec{a}\Delta t^2$

$\vec{a} = 9.8\ m/s^2\ [down]$ $\qquad$ $\Delta\vec{d} = \frac{1}{2}(9.8\ m/s^2)(0.50\ s)^2$

$\Delta\vec{d} = ?$ $\qquad$ $\Delta\vec{d} = 1.225\ m$

$\Delta t = 0.50\ s$ $\qquad$ $\Delta\vec{d} = 1.2\ m\ [down]$

$h = 1.5\ m - 1.2\ m = 0.3\ m$

c) What is the final velocity of the snowball?

From y direction:

$v_2^2 = v_1^2 + 2a\Delta d$

$v_2 = \sqrt{2a\Delta d}$

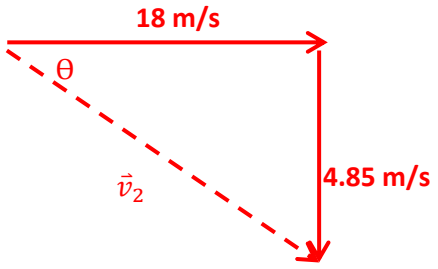
$v_2 = \sqrt{2(9.8\ m/s^2)(1.2\ m)}$

$v_2 = 4.85\ m/s$

Combining directions:

$\vec{v}_{2x} = 18\ m/s\ [fwd]$

$\vec{v}_{2y} = 4.85\ m/s\ [down]$



$v_2 = \sqrt{(v_{2x})^2 + (v_{2y})^2}$

$v_2 = \sqrt{(18\ m/s)^2 + (4.85\ m/s)^2}$ $\qquad$ $\tan(\theta) = \frac{4.85\ m/s}{18\ m/s}$

$v_2 = 18.64\ m/s$ $\qquad$ $\theta = \tan^{-1}\left(\frac{4.85\ m/s}{18\ m/s}\right)$

$v_2 = 19\ m/s$ $\qquad$ $\theta = 14°$

$\vec{v}_2 = 19\ m/s\ [fwd 14° down]$

8. A stone is thrown horizontally with an initial speed of 8.0 m/s from a cliff. Air resistance is negligible.

a) Determine the horizontal and vertical components of displacement at t = 1.0 s and 3.0 s.

x direction:

$\vec{v}_1 = 8.0\ m/s\ [fwd]$
$\vec{v}_2 = ?$
$\vec{a} = 0\ m/s^2\ [fwd]$
$\Delta\vec{d} = ?$
$\Delta t = 1.0\ s, 3.0\ s$

y direction:

$\vec{v}_1 = 0\ m/s\ [down]$
$\vec{v}_2 = ?$
$\vec{a} = 9.8\ m/s^2\ [down]$
$\Delta\vec{d} = ?$
$\Delta t = 1.0\ s, 3.0\ s$

t = 1.0 s

From x direction:

$\Delta\vec{d} = \vec{v}_1\Delta t + \frac{1}{2}\vec{a}\Delta t^2$
$\Delta\vec{d} = \vec{v}_1\Delta t$
$\Delta\vec{d} = (8.0\ m/s)(1.0\ s)$
$\Delta\vec{d} = 8.0\ m\ [fwd]$

From y direction:

$\Delta\vec{d} = \vec{v}_1\Delta t + \frac{1}{2}\vec{a}\Delta t^2$
$\Delta\vec{d} = \frac{1}{2}\vec{a}\Delta t^2$
$\Delta\vec{d} = \frac{1}{2}(9.8\ m/s^2)(1.0\ s)^2$
$\Delta\vec{d} = 4.9\ m\ [down]$

t = 3.0 s

From x direction:

$\Delta\vec{d} = \vec{v}_1\Delta t + \frac{1}{2}\vec{a}\Delta t^2$
$\Delta\vec{d} = \vec{v}_1\Delta t$
$\Delta\vec{d} = (8.0\ m/s)(3.0\ s)$
$\Delta\vec{d} = 24\ m\ [fwd]$

From y direction:

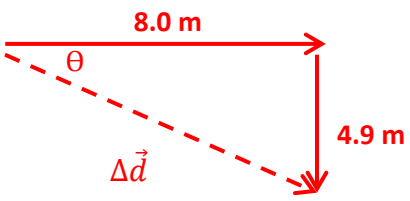
$\Delta\vec{d} = \vec{v}_1\Delta t + \frac{1}{2}\vec{a}\Delta t^2$
$\Delta\vec{d} = \frac{1}{2}\vec{a}\Delta t^2$
$\Delta\vec{d} = \frac{1}{2}(9.8\ m/s^2)(3.0\ s)^2$
$\Delta\vec{d} = 44\ m\ [down]$

b) Determine the total displacement at t = 1.0 s, and 3.0 s.

t = 1.0 s

Combining directions:

$\Delta\vec{d}_x = 8.0\ m\ [fwd]$
$\Delta\vec{d}_y = 4.9\ m\ [down]$



$$\Delta d = \sqrt{(v_{2x})^2 + (v_{2y})^2}$$

$$\Delta d = \sqrt{(8.0\ m)^2 + (4.9\ m)^2} \qquad \tan(\theta) = \frac{4.9\ m}{8.0\ m}$$

$$\Delta d = 9.381\ m \qquad \theta = \tan^{-1}\left(\frac{4.9\ m}{8.0\ m}\right)$$

$$\Delta d = 9.4\ m/s \qquad \theta = 31°$$

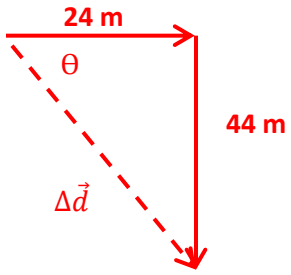
$$\Delta\vec{d} = \ 9.4\ m/s\ [fwd31°down]$$

t = 3.0 s

Combining directions:

$$\Delta\vec{d}_x = \ 24\ m\ [fwd]$$

$$\Delta\vec{d}_y = \ 44\ m\ [down]$$



$$\Delta d = \sqrt{(v_{2x})^2 + (v_{2y})^2}$$

$$\Delta d = \sqrt{(24\ m)^2 + (44\ m)^2} \qquad \tan(\theta) = \frac{44\ m}{24\ m}$$

$$\Delta d = 50.11\ m \qquad \theta = \tan^{-1}\left(\frac{44\ m}{24\ m}\right)$$

$$\Delta d = 5.0 \times 10^1\ m/s \qquad \theta = 61°$$

$$\Delta\vec{d} = \ 5.0 \times 10^1\ m/s\ [fwd61°down]$$

9. Explain why an airplane moving through the air is not an example of projectile motion.

An object considered a 'projectile' only has the force of gravity acting on it, and no other. An airplane moving through the air, while experiencing a force of gravity from the Earth, also experiences a force from the engine (to be accurate, it is actually a force from the particles the engine shoots backwards) that keeps it moving, so it would not be considered a projectile.