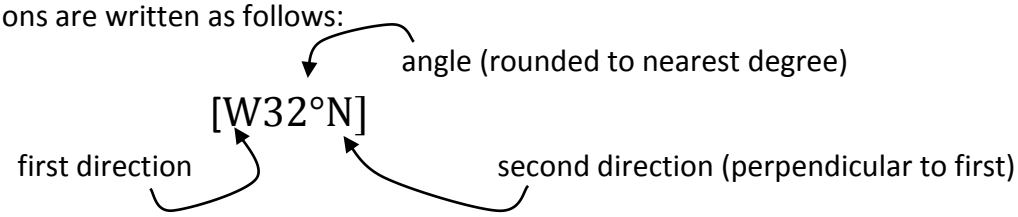
2-D Vectors

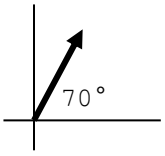
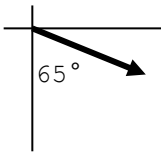
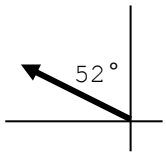
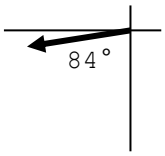
Solutions

Vector directions are written as follows:

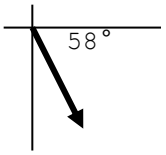
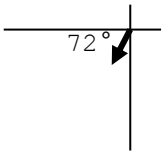
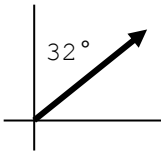
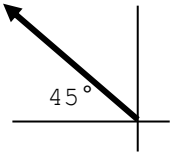


The above direction can be read as: "*starting West, rotate 32° towards the North*"

Examples:

Vector				
Direction (option 1)	[N20°E]	[E25°S]	[N52°W]	[S84°W]
Direction (option 2)	[E70°N]	[S65°E]	[W38°N]	[W6°S]

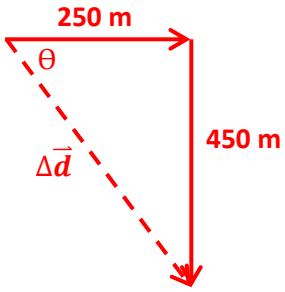
Practice:

Vector				
Direction (option 1)	**[E58°S]**	**[W72°S]**	**[N32°E]**	**[W45°N]**
Direction (option 2)	**[S32°S]**	**[S18°W]**	**[E58°N]**	**[N45°W]**

2-D Displacement Problems:

1. a) Sketch and calculate the total displacement of

i) Ms. Reichling who travels 250 m [E] and then 450 m [S]



$\Delta d = \sqrt{(250\ m)^2 + (450\ m)^2}$

$\Delta d = 514.8\ m$

$\Delta d = 510\ m$

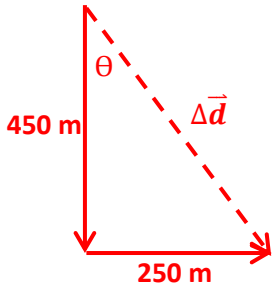
$\tan(\theta) = \frac{450\ m}{250\ m}$

$\theta = \tan^{-1}\left(\frac{450\ m}{250\ m}\right)$

$\theta = 61°$

$\Delta\vec{d} = 510\ m\ [E61°S]$

ii) Ms. McTague who travels 450 m [S] and then 250 m [E].



$\Delta d = \sqrt{(450\ m)^2 + (250\ m)^2}$ $\quad \tan(\theta) = \frac{250\ m}{450\ m}$

$\Delta d = 514.8\ m$ $\quad \theta = \tan^{-1}\left(\frac{250\ m}{450\ m}\right)$

$\Delta d = 510\ m$ $\quad \theta = 29°$

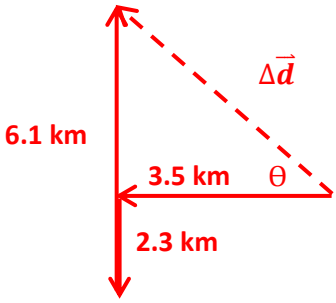
$\Delta\vec{d} = 510\ m\ [S29°E]$

b) If Ms.McTague were to turn around and head directly back to where she started from, in what direction would she be travelling?

(Z-pattern)

[N29°W]

2. Find the total displacement of a cyclist who travels 3.5 km [W], 2.3 km[S], then 6.1 km [N].



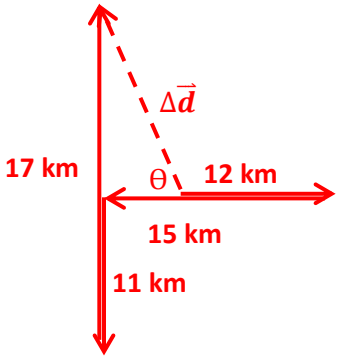
$\Delta d = \sqrt{(3.5\ km)^2 + (3.8\ km)^2}$ $\quad \tan(\theta) = \frac{3.8\ km}{3.5\ km}$

$\Delta d = 5.167\ km$ $\quad \theta = \tan^{-1}\left(\frac{3.8\ km}{3.5\ km}\right)$

$\Delta d = 5.2\ km$ $\quad \theta = 47°$

$\Delta\vec{d} = 5.2\ km\ [W47°N]$

3. Find the total displacement of a bird that travels 12 km [E], 15 km [W], 11 km[S], then 17 km [N].



$\Delta d = \sqrt{(3.0\ m)^2 + (6.0\ m)^2}$ $\quad \tan(\theta) = \frac{6.0\ km}{3.0\ km}$

$\Delta d = 6.71\ km$ $\quad \theta = \tan^{-1}\left(\frac{6.0\ km}{3.0\ km}\right)$

$\Delta d = 6.7\ km$ $\quad \theta = 63°$

$\Delta\vec{d} = 6.7\ km\ [W63°N]$

4. Find the total displacement of a fish that swims 320 m [N], 85 m [down], 110 m[N], then 45 m [up].

$\Delta d = \sqrt{(430\ m)^2 + (40\ m)^2}$

$\Delta d = 431.9\ m$

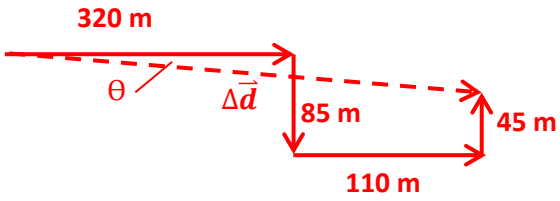
$\Delta d = 430\ m$

$\tan(\theta) = \frac{40\ m}{430\ m}$

$\theta = \tan^{-1}\left(\frac{40\ m}{430\ m}\right)$

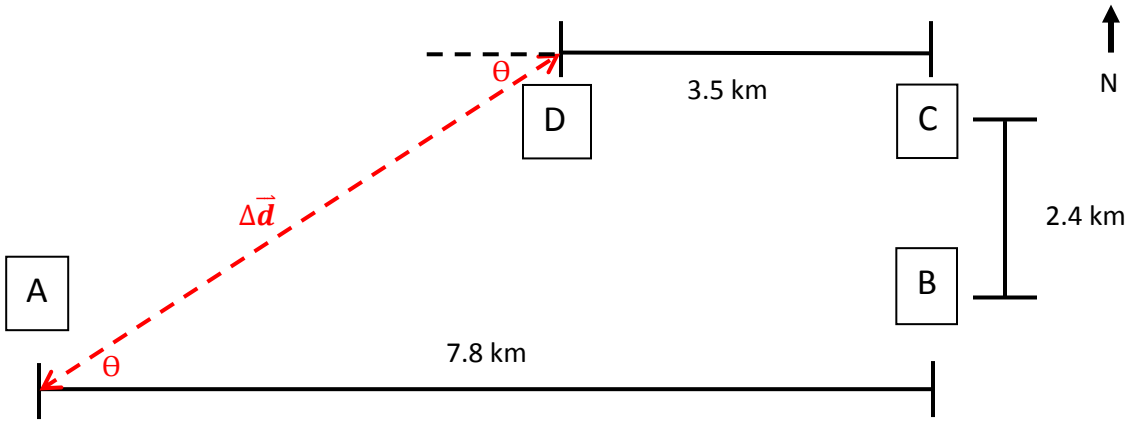
$\theta = 5°$

$\Delta\vec{d} = 430\ m\ [N5°down]$



2-D Vector Problems:

5. A group of hikers sets out from point ***A***, proceeds to ***B***, then to ***C***, and finally to ***D***. The entire trip takes 6.0 h.



a) Determine the hikers' average speed for the trip.

$v = \frac{d}{t}$

$v = \frac{7.8\ km + 2.4\ km + 3.5\ km}{6.0\ h}$

$v = 2.28\ km/h$

$v = 2.3\ km/h$

b) If the hikers release a homing pigeon upon their arrival at point ***D*** and the bird returns to point ***A*** 30 min later, what is the bird's average velocity during the flight?

$\Delta d = \sqrt{(4.3\ km)^2 + (2.4\ km)^2}$

$\Delta d = 4.92\ km$

$\Delta d = 4.9\ km$

$\tan(\theta) = \frac{2.4\ km}{4.3\ km}$

$\theta = \tan^{-1}\left(\frac{2.4\ km}{4.3\ km}\right)$

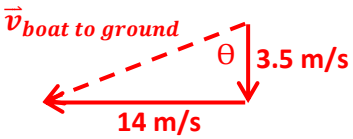
$\theta = 29°$

$$\vec{v} = \frac{\Delta \vec{d}}{\Delta t}$$
$$v = \frac{4.9\ km\ [W29°S]}{0.5\ h}$$
$$v = 9.8\ km/h\ [W29°S]$$

c) Determine the hikers' average velocity for the trip.

$$\vec{v} = \frac{\Delta \vec{d}}{\Delta t}$$
$$v = \frac{4.9\ km\ [E79°N]}{6.0\ h}$$
$$v = 0.82\ km/h\ [E29°N]$$

6. A boat that can travel at 14 m/s in still water is crossing a river that is moving at 3.5 m/s [S]. If the boat always points itself west, in what direction will the boat appear to be moving to a person standing on the shore?

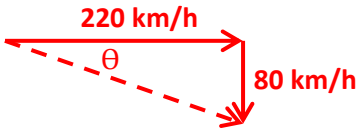


$$\tan(\theta) = \frac{14\ m/s}{3.5\ m/s}$$
$$\theta = \tan^{-1}\left(\frac{14\ m/s}{3.5\ m/s}\right)$$
$$\theta = 76°$$

Direction of boat according to shore = [S76°W]

7. A plane leaves Toronto and flies with an airspeed of 2.20 x 10^2 km/h always pointing due east. A wind is blowing from the north at 8.0 x 10^1 km/h.

a) What is the plane's velocity relative to the ground?



$$v_{pg} = \sqrt{(220\ km/h)^2 + (80\ km/h)^2}$$
$$v_{pg} = 234.1\ km/h$$
$$v_{pg} = 230\ km/h$$

$$\tan(\theta) = \frac{80\ km/h}{220\ km/h}$$
$$\theta = \tan^{-1}\left(\frac{80\ km/h}{220\ km/h}\right)$$
$$\theta = 20°$$

$$\vec{v}_{pg} = 230\ km/h\ [E20°S]$$

b) What is the plane's displacement from Toronto after flying for 2.5 h?

$$\Delta \vec{d} = \vec{v}\Delta t$$
$$\Delta \vec{d} = (230\ km/h\ [E20°S])(2.5\ h)$$
$$\Delta \vec{d} = 575\ km\ [E20°S]$$
$$\Delta \vec{d} = 580\ km\ [E20°S]$$

8. A boat sets out from the north shore of a 200 m wide east-flowing river. The boat always faces due south but the current carries it 300 m downstream while crossing. The trip takes 2.0 min. Assume three significant digits.

   a) What is the boat's displacement during the trip?

$\Delta d = \sqrt{(200\ m)^2 + (300\ m)^2}$

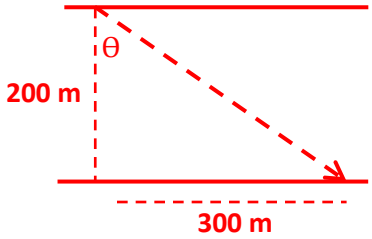
$\Delta d = 360.6\ m$

$\Delta d = 361\ m$

$\tan(\theta) = \frac{300\ m}{200\ m}$

$\theta = \tan^{-1}\left(\frac{300\ m}{200\ m}\right)$

$\theta = 56°$



$\Delta\vec{d} = 361\ m\ [S56°E]$

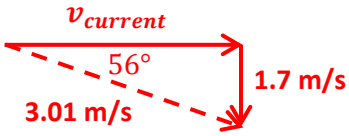
   b) What is the boat's average velocity during the trip?

$\vec{v} = \frac{\Delta\vec{d}}{\Delta t}$

$v = \frac{361\ m\ [S56°E]}{120\ s}$

$v = 3.01\ m/s\ [S56°E]$

   c) If the boat's velocity relative to the water is 1.7 m/s [S], what is the velocity of the current?



$v_{current} = \sqrt{(3.01\ m/s)^2 - (1.7\ m/s)^2}$

$v_{current} = 2.484\ m/s$

$v_{current} = 2.5\ m/s\ [E]$