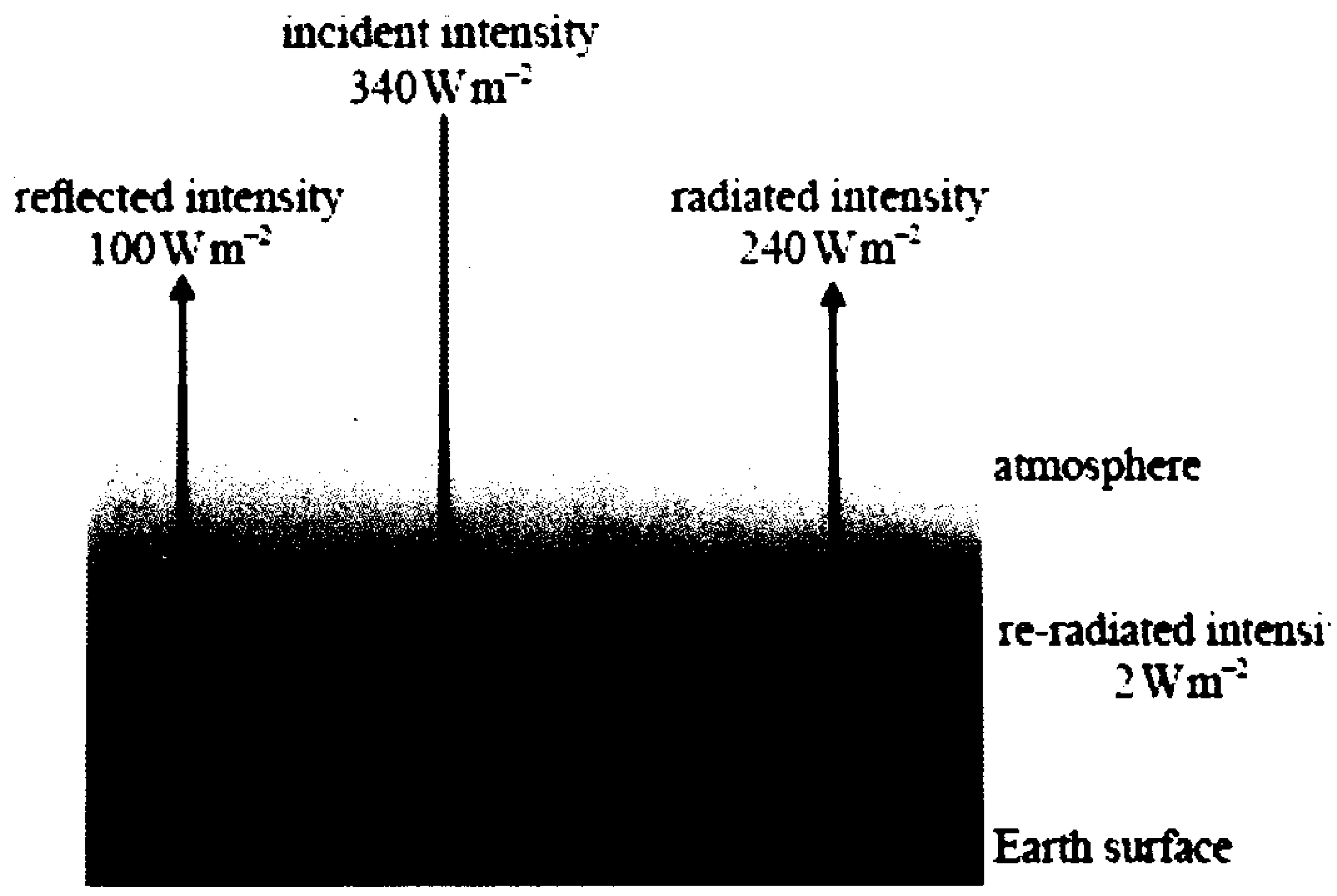
8.5 GREENHOUSE EFFECT
8.6 GLOBAL WARMING

HW/Study Packet SOLUTIONS

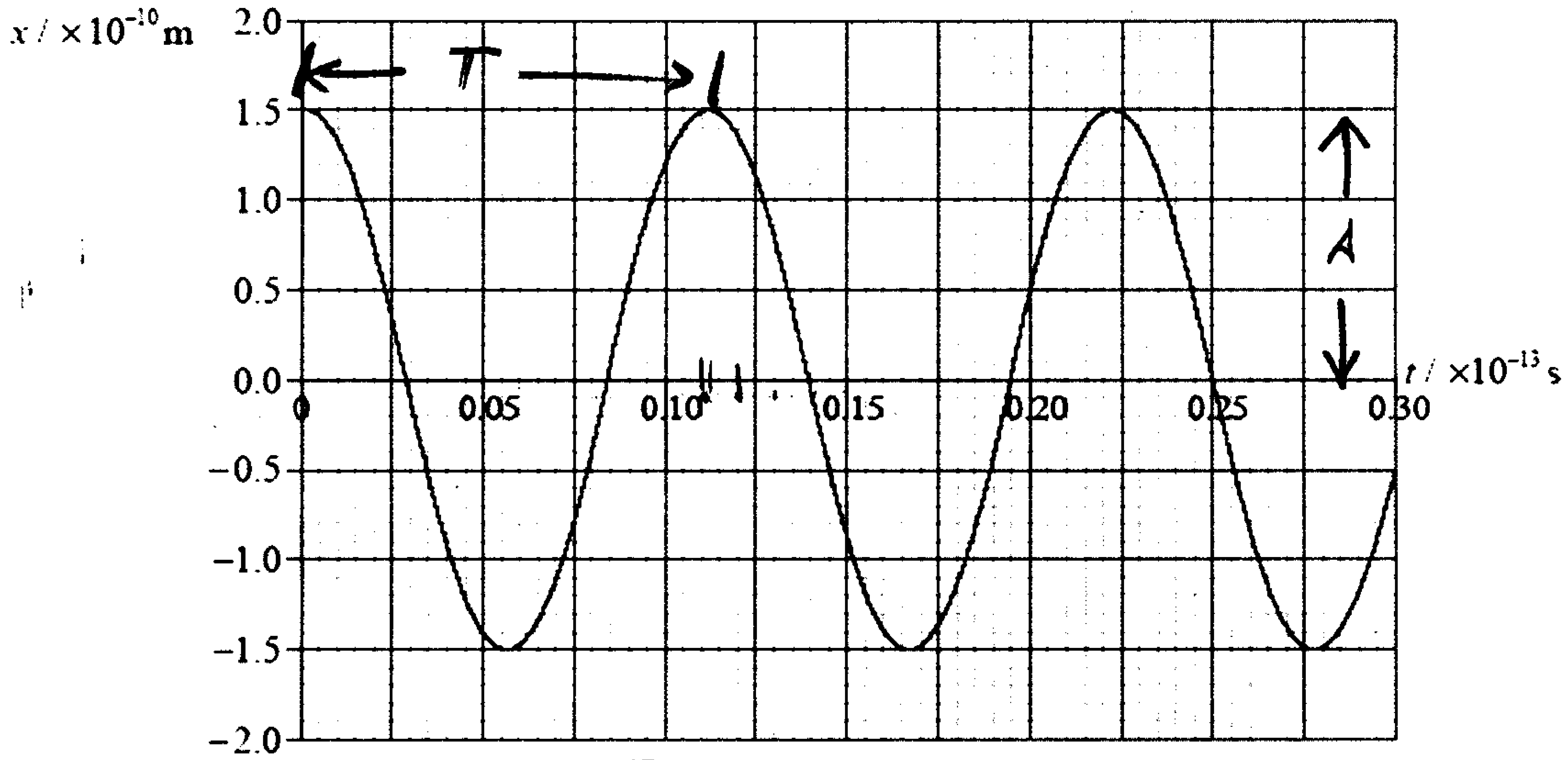
1. The diagram shows a simplified model of the energy balance for Earth. Determine the albedo of the Earth according to this model.



$$\alpha = \frac{\text{REFLECTED POWER}}{\text{INCIDENT POWER}}$$

$$= \frac{100}{340} = 29.4\% = \boxed{30\%}$$

2. In a simple model of a methane molecule, a hydrogen atom and the carbon atom can be regarded as two masses attached by a spring. A hydrogen atom is much less massive than the carbon atom such that any displacement of the carbon atom may be ignored. The graph below shows the variation with time *t* of the displacement *x* from its equilibrium position of a hydrogen atom in a molecule of methane.



The mass of hydrogen atom is 1.7 x 10^{-27} kg. Use data from the graph to determine:

a) its amplitude of oscillation. FROM THE GRAPH, $A = \boxed{1.5 \times 10^{-10}\ \text{m}}$

b) the frequency of its oscillation.

FROM THE GRAPH, $T = 0.11 \times 10^{-13}\ \text{s}$; $f = \frac{1}{T} = \frac{1}{0.11 \times 10^{-13}} = \boxed{9.1 \times 10^{13}\ \text{Hz}}$

c) the maximum kinetic energy of the hydrogen atom.

ω = ANGULAR FREQUENCY $= 2\pi f = 2\pi(9.1 \times 10^{13}) = 5.712 \times 10^{14}\ \text{rad s}^{-1}$.

$$E_{K_{MAX}} = \frac{1}{2} m\omega^2 x_0^2 = \frac{1}{2}(1.7 \times 10^{-27})(5.712 \times 10^{14})^2(1.5 \times 10^{-10})^2$$

$$= \boxed{6.2 \times 10^{-18}\ \text{J}}$$

Assuming that the motion of the hydrogen atom is simple harmonic, its frequency of oscillation f is given by the expression

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m_p}},$$

where k is the force per unit displacement between a hydrogen atom and the carbon atom and m_p is the mass of a proton.

d) Determine the value of k.

$$f = \frac{1}{2\pi}\left(\frac{k}{M_p}\right)^{1/2} \Rightarrow (2\pi f)^2 M_p = 4\pi^2(9.1\times10^{13})^2(1.7\times10^{-27})$$
$$= 555.8\ Nm^{-1} = \boxed{560\ Nm^{-1}}$$

e) Estimate the maximum acceleration of the hydrogen atom.

TREAT THE MOTION AS "SPRING-LIKE";

$$F = kx = ma \Rightarrow a = \frac{kx}{m} = \frac{(555.8)(1.5\times10^{-10})}{1.7\times10^{-27}} = \boxed{5.0\times10^{19}\ ms^{-2}}$$

f) Electromagnetic radiation of frequency 9.1 x 10^{13} Hz is in the infrared region of the electromagnetic spectrum. Suggest, based on your answer to part (b), why methane is classified as a 'greenhouse gas'.

For a CH_4 molecule, we have found that the resonant frequency is 9.1×10^{13} Hz (part b). Radiation of the same frequency will be readily absorbed by the CH_4 molecule due to resonance; therefore the CH_4 molecule will acquire more energy easily. Therefore, CH_4 = "Greenhouse gas"

3. A copper ball 2.0 cm in radius is heated in a furnace to 4.0 x 10^2°C. If its emissivity is 0.30, at what rate does it radiate energy? → (673 K)

$$A = 4\pi r^2 = 4\pi(0.02)^2 = 0.005\ m^2$$

r = 0.02 m.

$$P = e\sigma AT^4 = (0.30)(5.67\times10^{-8})(0.005)(673)^4$$
$$= 17.45\ W = \boxed{17\ W}$$

4. The sun radiates energy at the rate of 6.5 x 10^7 Wm^{-2} from its surface. Assuming that the sun radiates as a blackbody, find its surface temperature.

$$P = e\sigma AT^4 \Rightarrow T = \left[\frac{P}{e\sigma A}\right]^{1/4} = \left[\frac{6.5\times10^7}{(1)(5.67\times10^{-8})}\right]^{1/4} = 5819\ K$$
$$= \boxed{5800\ K}$$

$$A_{SUN} = 4\pi r^2 = 4\pi(7.0\times10^8)^2$$
$$= 6.158\times10^{18}\ m^2$$

which is irrelevant in this problem (why ?!?) :)

5. The following data relate to the Earth and the Sun:

Earth-Sun distance	$= 1.5 \times 10^{11}$ m
radius of Earth	$= 6.4 \times 10^{6}$ m
radius of Sun	$= 7.0 \times 10^{8}$ m
surface temperature of Sun	$= 5800$ K

a) Using data from the table, calculate the power radiated by the Sun.

$$P = e\sigma A T^4 = (1)(5.67\times10^{-8})[4\pi(7.0\times10^{8})^2](5800)^2$$
$$= 3.95\times10^{26}\,\text{W} = \boxed{4.0\times10^{26}\,\text{W}}$$

b) Calculate the solar power incident per unit area at a distance from the Sun equal to the Earth's distance from the Sun.

$$\frac{\text{TOTAL POWER}}{\text{SPHERICAL AREA}} = \frac{3.95\times10^{26}}{4\pi(1.5\times10^{11})^2} = 1397\ \text{Wm}^{-2} = \boxed{1400\ \text{Wm}^{-2}}$$

(P/A)

c) Show that the value for power absorbed per unit area of 240 W m^{-2} (for the Earth) is consistent with an average equilibrium temperature for Earth of about 255 K.

We treat Earth as a blackbody, so Power absorbed = Power radiated...

$$P_{IN} = P_{OUT} \Rightarrow \frac{P}{A} = e\sigma T^4 \Rightarrow T = \left[\left(\frac{P}{A}\right)\left(\frac{1}{e\sigma}\right)\right]^{1/4} = \boxed{255\ K} \checkmark$$

d) Explain, by reference to the greenhouse effect, why the average temperature of the surface of the Earth is greater than 255 K.

Incoming solar radiation is absorbed by Earth & re-emitted at longer λ's (IR); longer λ's are strongly absorbed by greenhouse gases (CO_2, CH_4, N_2O, etc...); some of this is re-radiated back to Earth's surface.

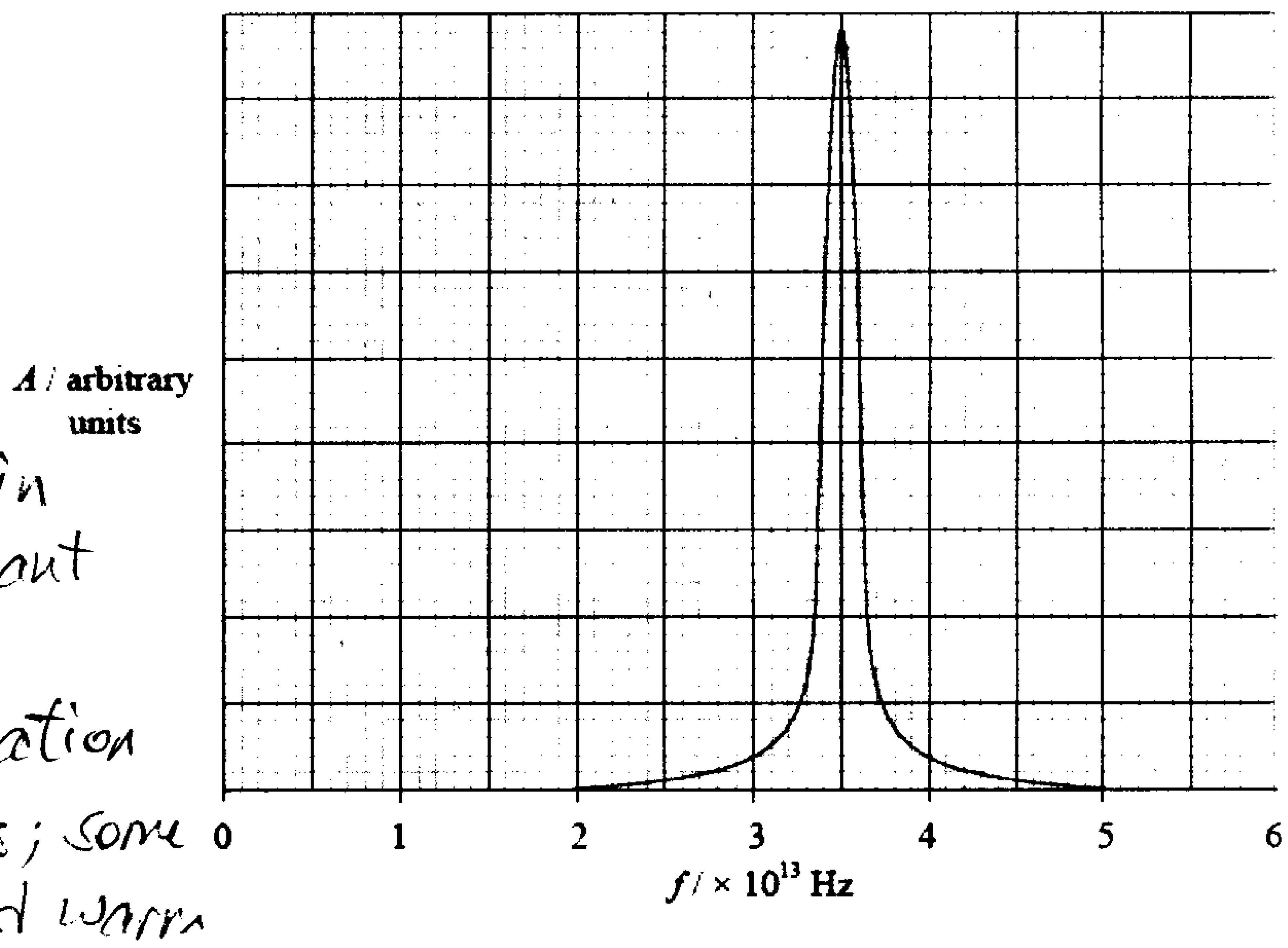
6. The graph shows part of the absorption spectrum of nitrogen oxide (N_2O) in which the intensity of absorbed radiation *A* is plotted against frequency *f*.

a) State the region of the electromagnetic spectrum to which the resonant frequency of nitrogen oxide belongs.

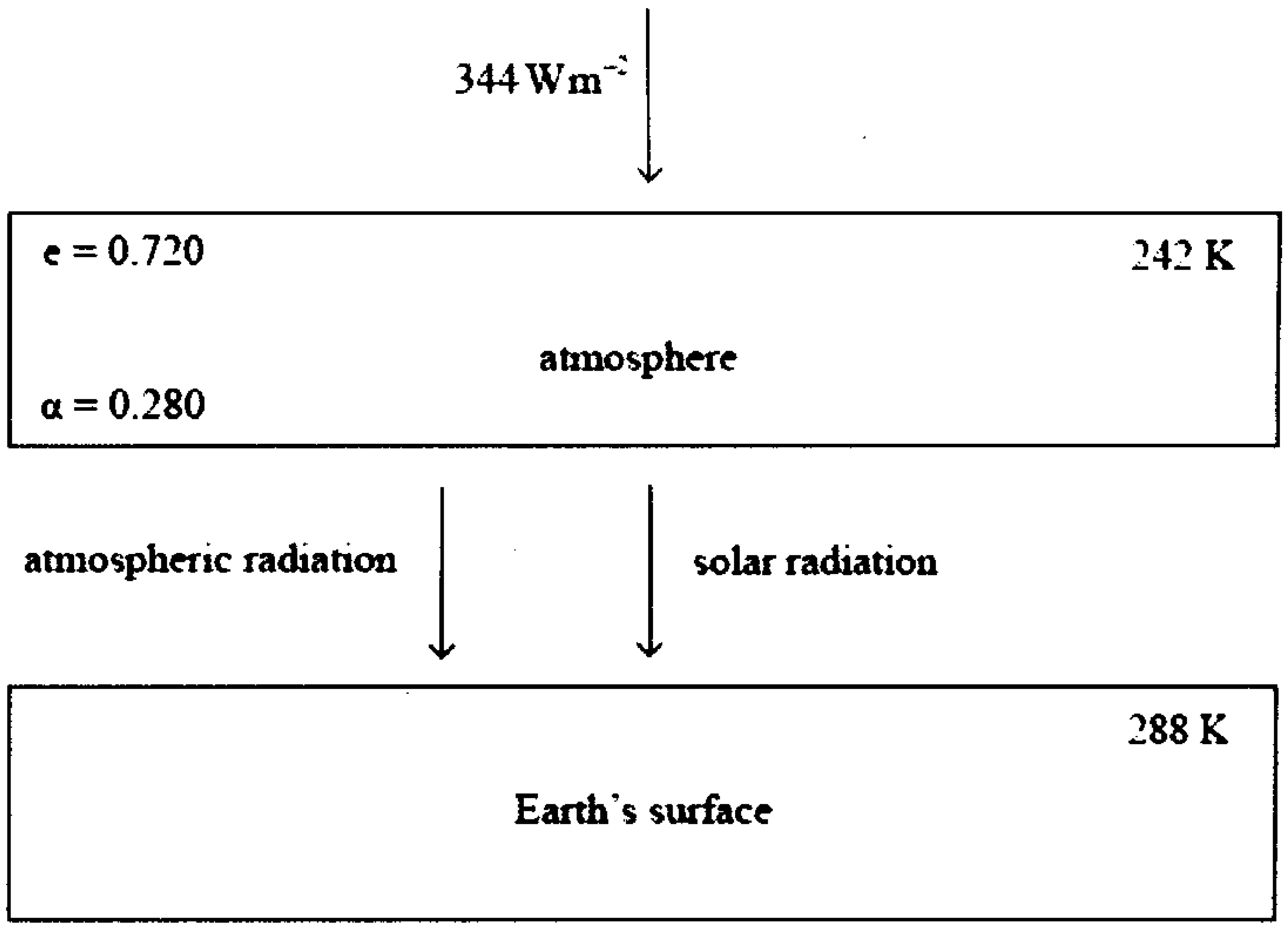
3.5×10^{13} Hz $\Rightarrow$ $\boxed{\text{IR REGION}}$

b) Using your answer to (a), explain why nitrogen oxide is classified as a greenhouse gas.

N_2O strongly absorbs radiation in the IR region because its resonant frequency matches it.
Therefore it will absorb IR radiation and re-emit it in all directions; some will get back to the surface and warm it.



c) The diagram shows a simple energy balance climate model in which the atmosphere and the surface of Earth are two bodies each at constant temperature. The surface of the Earth receives both solar radiation and radiation emitted from the atmosphere. Assume that the Earth's surface behaves as a black body.



The following data are available for this model:

average temperature of the atmosphere of Earth	*= 242 K*
emissivity, e of the atmosphere of Earth	*= 0.720*
average albedo, α of the atmosphere of Earth	*= 0.280*
solar intensity at top of atmosphere	*= 344 W m^{-2}*
average temperature of the surface of Earth	*= 288 K*

Determine:

d) The power radiated per unit area of the atmosphere.

$$\frac{P}{A} = e\sigma T^4 = (0.720)(5.67\times10^{-8})(242)^4 = \boxed{140\,\mathrm{Wm^{-2}}}$$

e) The solar power absorbed per unit area at the surface of the Earth.

$$\left(\frac{P}{A}\right)_{\text{surface}} = (1-\alpha)\left(\frac{P}{A}\right)_{\substack{\text{Top}\\ \text{of}\\ \text{atmosphere}}} = (1-0.280)(344) = \boxed{248\,\mathrm{Wm^{-2}}}$$

It is hypothesized that, if the production of greenhouse gases were to stay at its present level then the temperature of the Earth's atmosphere would eventually rise by 6.0 K.

f) Calculate the power per unit area that would then be radiated by the atmosphere.

$$\frac{P}{A} = e\sigma T^4 = (1)(5.67\times10^{-8})(\underset{(242+6)}{\underset{\uparrow}{248}})^4 = \boxed{154\,\mathrm{Wm^{-2}}}$$

g) Calculate the power per unit area that would then be absorbed by the Earth's surface.

$$\left(\frac{P}{A}\right)_{\text{Surface}} + \left(\frac{P}{A}\right)_{\substack{\text{radiated}\\ \text{by}\\ \text{atmosphere}}} = 248 + 154 = \boxed{402\,\mathrm{Wm^{-2}}}$$

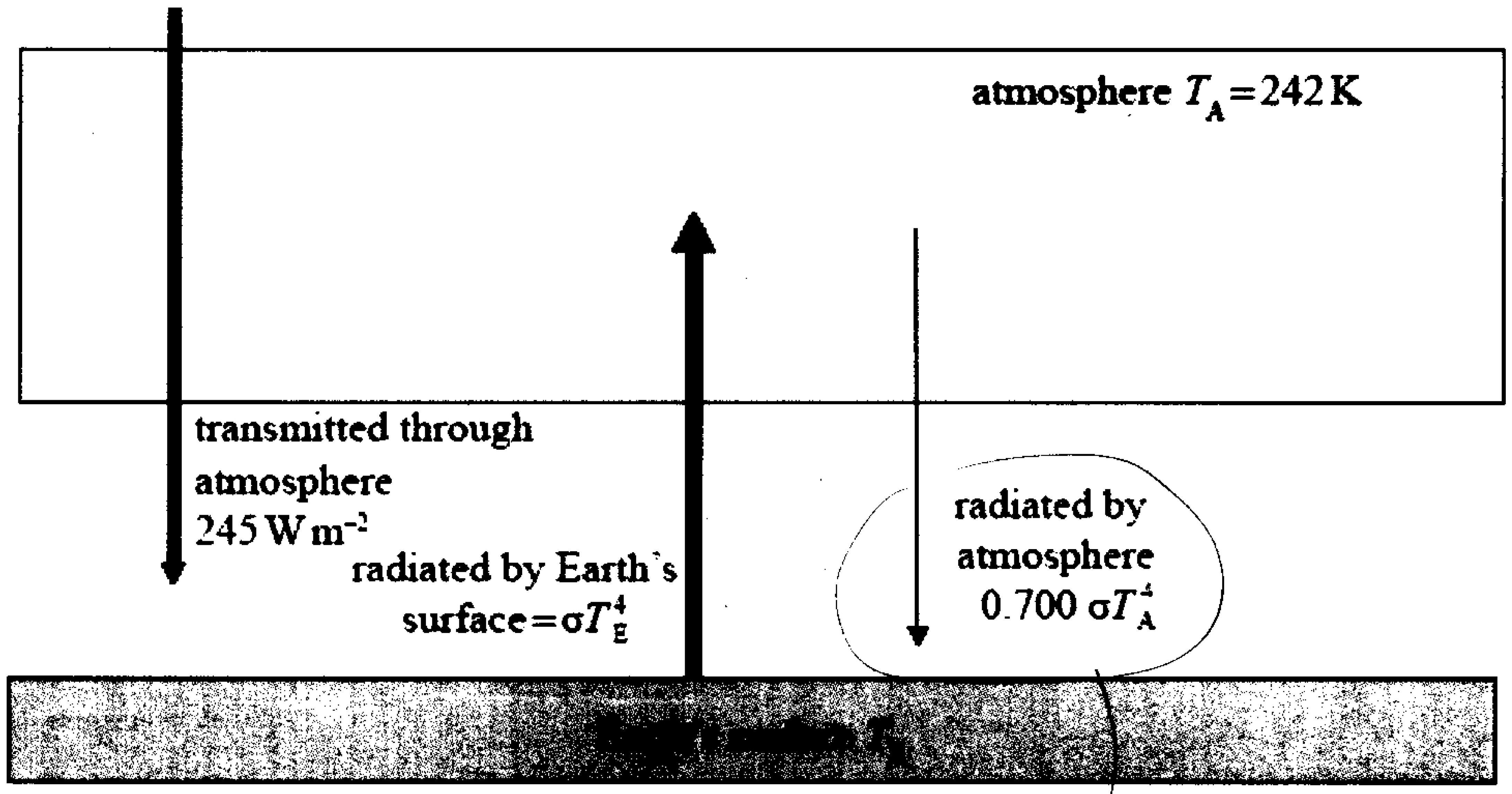
h) Estimate the increase in temperature of Earth's surface.

$$\frac{P}{A} = e\sigma T^4 \Rightarrow T = \left[\left(\frac{P}{A}\right)\frac{1}{e\sigma}\right]^{1/4} = \left[\frac{(402)}{5.67\times10^{-8}}\right]^{1/4} = 290\,\mathrm{K}$$

$$\therefore \Delta T = 290 - 288 = \boxed{+2\,\mathrm{K}}$$

7. The diagram shows a simplified model of the energy balance of the Earth's surface. The diagram shows radiation entering or leaving the Earth's surface only.

The average equilibrium temperature of the Earth's surface is T_E and that of the atmosphere is T_A = 242 K.



a) Using the data from the diagram, state the emissivity of the atmosphere.

Since $P = e\sigma T^4$, $\boxed{e = 0.700}$

b) Determine the intensity of the radiation radiated by the atmosphere towards the Earth's surface.

$$I = \frac{P}{A} = \frac{e\sigma A T_A^4}{A} = e\sigma T_A^4 = (0.700)(5.67\times10^{-8})(242)^4$$
$$= \boxed{136\ \mathrm{W\,m^{-2}}}$$

c) Calculate T_E.

TOTAL RADIATION PER m^2 = RADIATION ABSORBED FROM ATMOSPHERE + RADIATION ABSORBED FROM ATMOSPHERE $= 245 + 136 = e\sigma T_E^4$

$$T_E = \left(\frac{381}{(1)(5.67\times10^{-8})}\right)^{1/4} = \boxed{286\ K}$$

8. One effect of global warming is to melt the Antarctic ice sheet. The following data are available for the Antarctic ice sheet and the Earth's oceans.

Area of ice sheet	*= 1.4 × 10^7 km^2* = 1.4×10^{13} m^2
Average thickness of ice	*= 1.5 × 10^3 m*
Density of ice	*= 920 kg m^{-3}*
Density of water	*= 1000 kg m^{-3}*
Area of Earth's oceans	*= 3.8 × 10^8 km^2*

$1\ km^2 = 10^6\ m^2$

Volume = AREA x THICKNESS.

Using the data, determine:

a) the mass of the Antarctic ice.

TOTAL VOLUME OF ICE = $(1.4\times10^{13})(1.5\times10^3) = 2.1\times10^{16}\ m^3$

$$\rho = \frac{m}{V} \Rightarrow m = \rho V = (920)(2.1\times10^{16}) = 1.932\times10^{19}\ kg = \boxed{1.9\times10^{19}\ kg}$$

b) the change in mean sea level if all the Antarctic ice sheet were to melt and flow into the oceans.

FOR H_2O: $\rho = \frac{M}{V} \Rightarrow V = \frac{m}{\rho} = \frac{(1.9\times10^{19})}{1000} = 1.9\times10^{16}\ m^3$ = Volume of H_2O from ice that melted...

FOR OCEAN $\Rightarrow$ VOLUME OF CHANGE = (AREA)(DEPTH) $\Rightarrow$ DEPTH = $\frac{\text{VOLUME}}{\text{AREA}}$

$$= \frac{1.9\times10^{16}}{3.8\times10^8} = \boxed{50\ m}$$

!! WOW !!