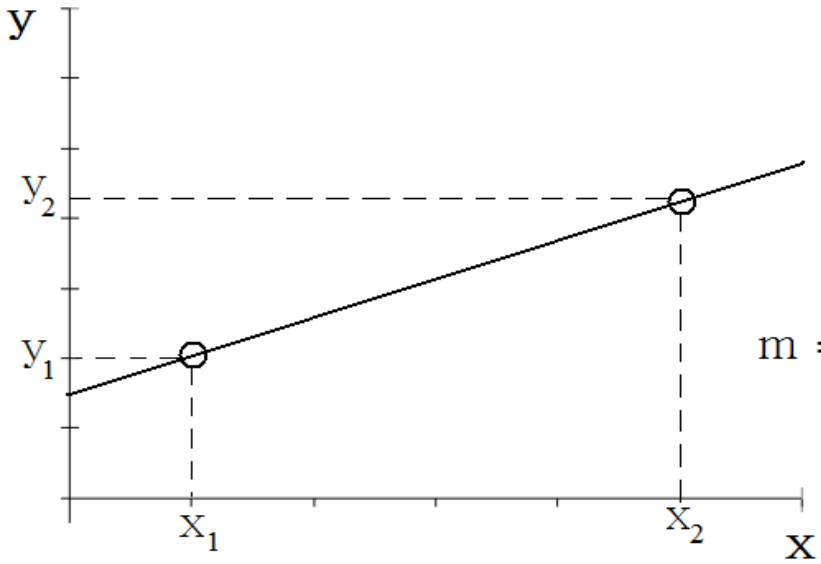
Unit I – Kinematics

- Kinematics Graphing

Position vs Time Graphs - Uniform Motion

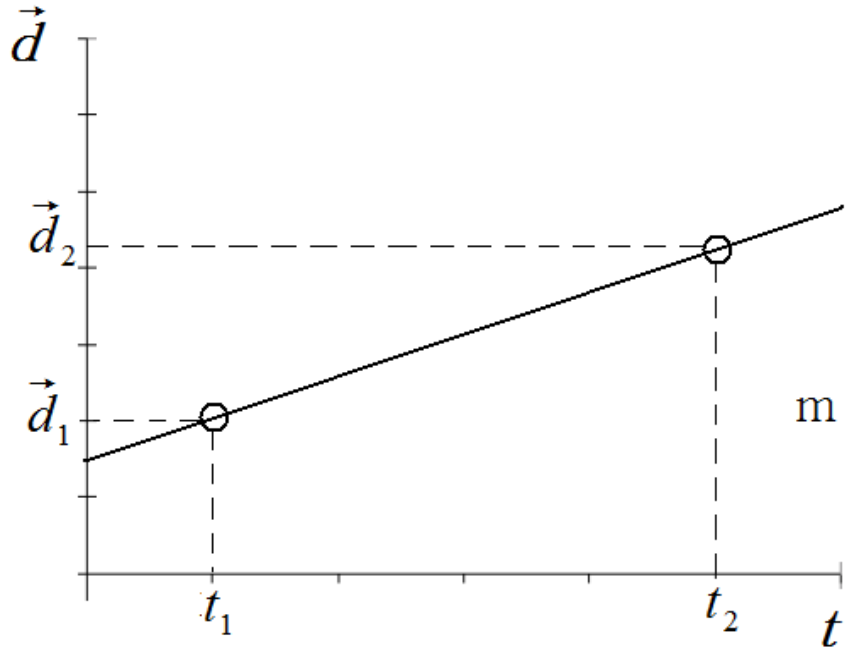
Math Review: Slope

To find the slope of a line, choose two points on the line and enter them into the equation below:



$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$$

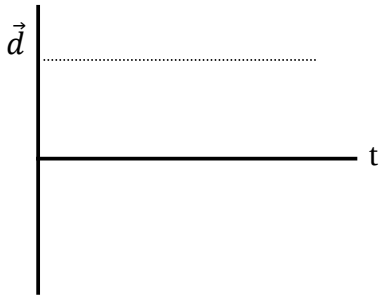
If we were to replace *y* with $\vec{d}$ and *x* with *t*, we would then get:



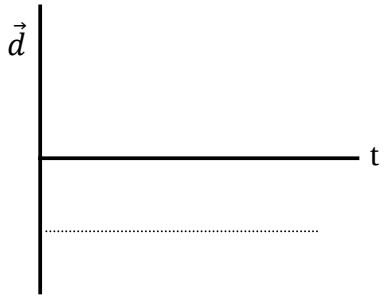
$$m = \frac{\vec{d}_2 - \vec{d}_1}{t_2 - t_1} = \frac{\Delta \vec{d}}{\Delta t} = \vec{v}$$

Therefore, the slope of a position vs time graph is velocity!!!

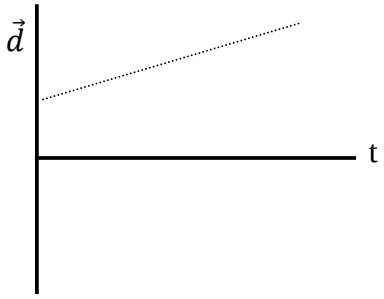
Five (Six) Possible position vs time graphs of uniform motion:



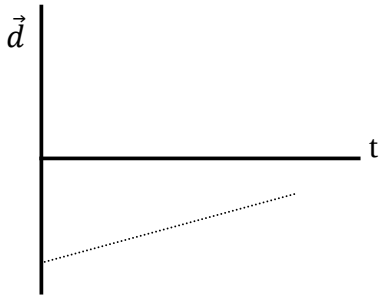
at rest, ahead of origin



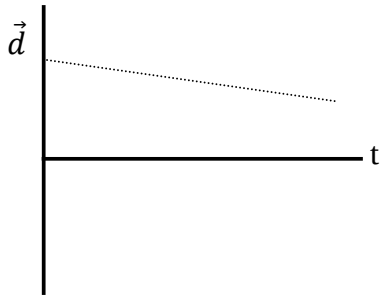
at rest, behind origin



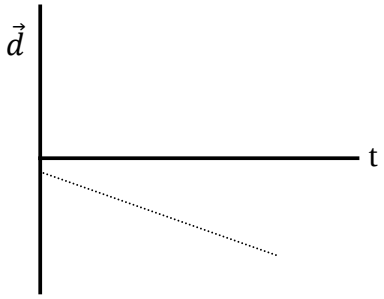
constant positive velocity, moving away from the origin



constant positive velocity, moving towards the origin



constant negative velocity, moving towards the origin



constant negative velocity, moving away from the origin

Non-Uniform Motion

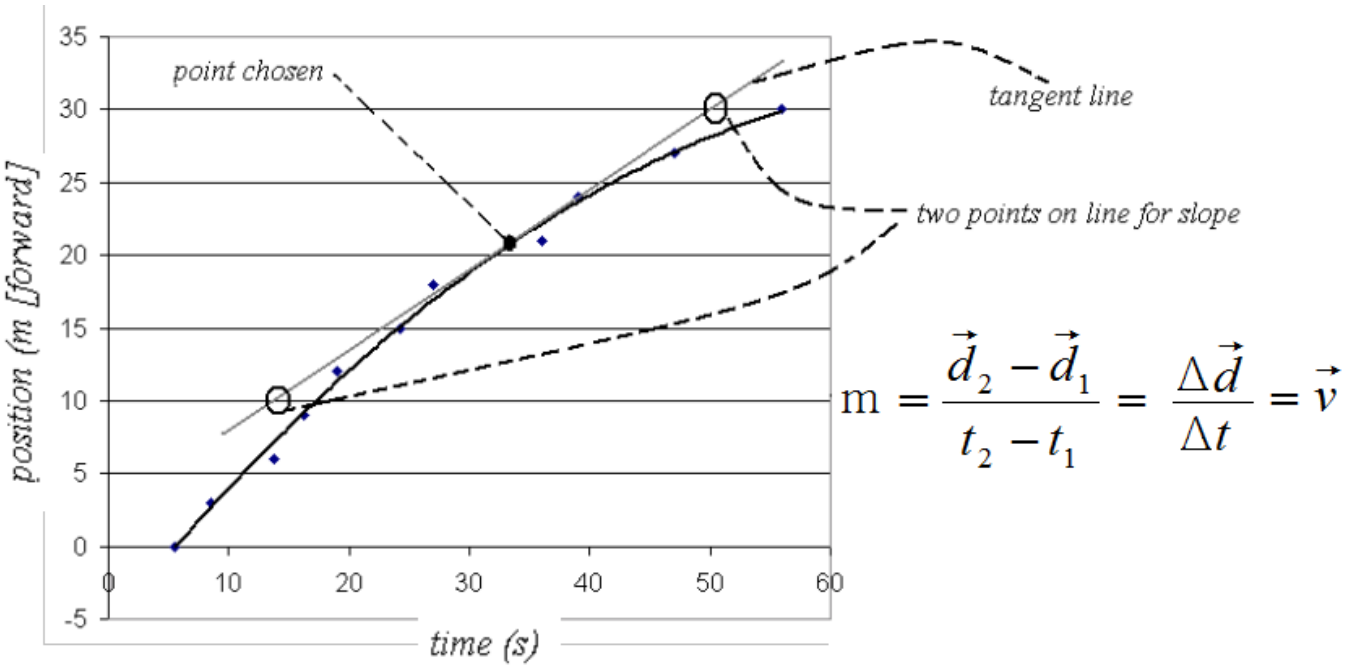
If an object is accelerating, you cannot inquire about its velocity, because its velocity is always changing. What you can inquire about is its velocity at one instant in time, or, its *instantaneous velocity*.

This means that on a position vs time graph, the slope is always changing, and so to find the instantaneous velocity of the object, you need to draw a tangent line:

Step 1: choose a point on the curve at the time you want to know the velocity for (*point chosen*)

Step 2: draw a line that only touches the curve at this point, which shows its curvature (*tangent line*)

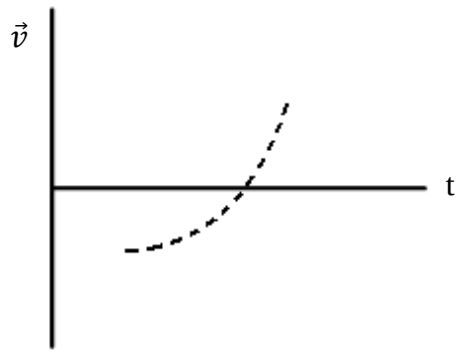
Step 3: Find the slope of your drawn tangent line to find the object's instantaneous velocity (*two points on line for slope*)



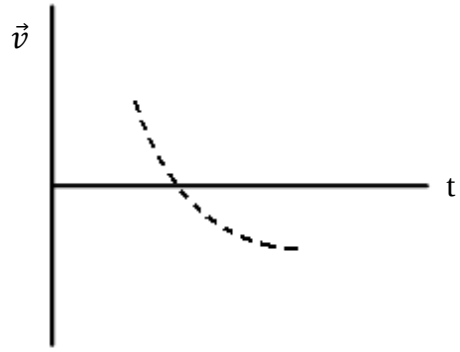
$$m = \frac{\vec{d}_2 - \vec{d}_1}{t_2 - t_1} = \frac{\Delta \vec{d}}{\Delta t} = \vec{v}$$

<u>Four Possible position vs time graphs of non-uniform motion:</u>

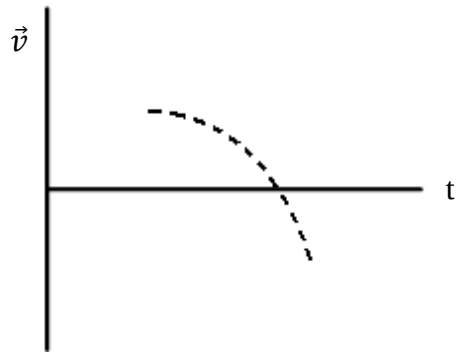
* is the slope negative or positive? → negative or positive direction
* is the magnitude of the slope increasing or decreasing (more or less steep?) → speeding up or slowing down



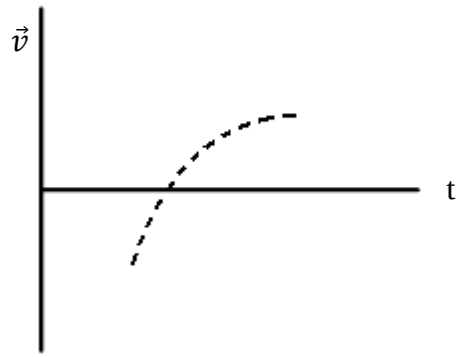
speeding up, positive direction



slowing down, negative direction



speeding up, negative direction



slowing down, positive direction

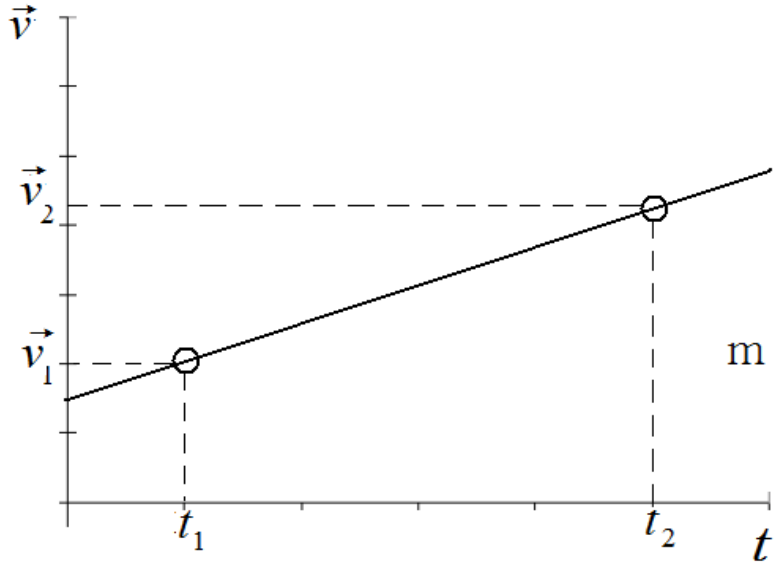
Recall:

speeding up, positive direction = positive acceleration
(+) (+) = (+)
slowing down, negative direction = positive acceleration
(−) (−) = (+)

speeding up, negative direction = negative acceleration
(+) (−) = (−)
slowing down, positive direction = negative acceleration
(−) (+) = (−)

<u>Velocity vs Time Graphs – Uniform & Non-Uniform Motion</u>

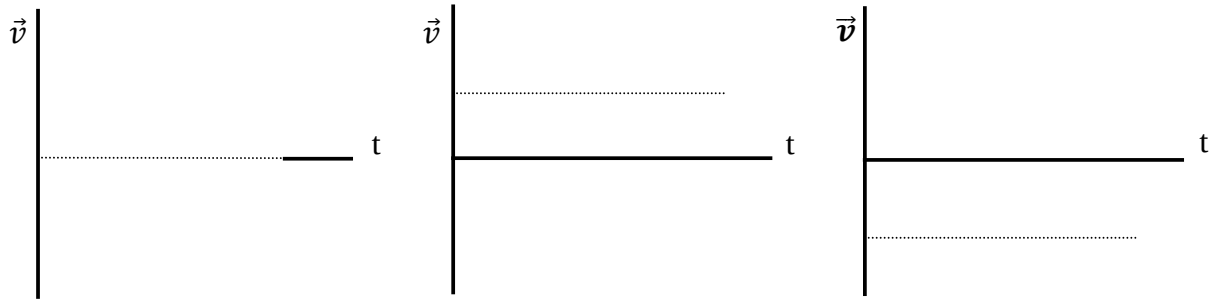
If we were to replace *y* with $\vec{v}$ and *x* with *t*, we would then get:



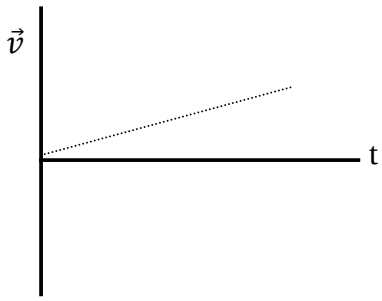
$$\text{m} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{\Delta \vec{v}}{\Delta t} = \vec{a}$$

Therefore, the slope of a velocity vs time graph is acceleration!!!

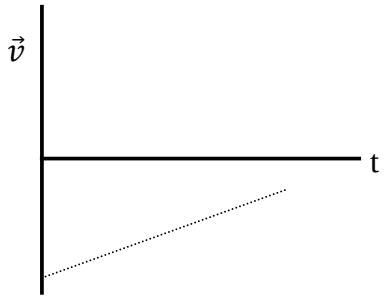
<u>Seven Possible velocity vs time graphs:</u>



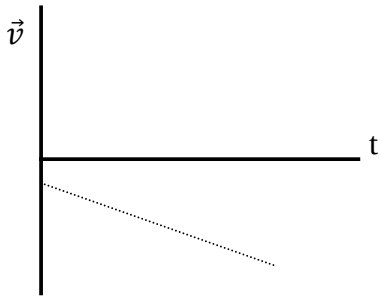
at rest constant positive velocity constant negative velocity



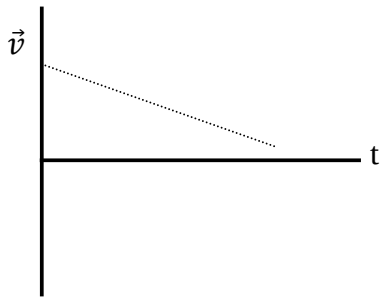
speeding up, positive direction



slowing down, negative direction



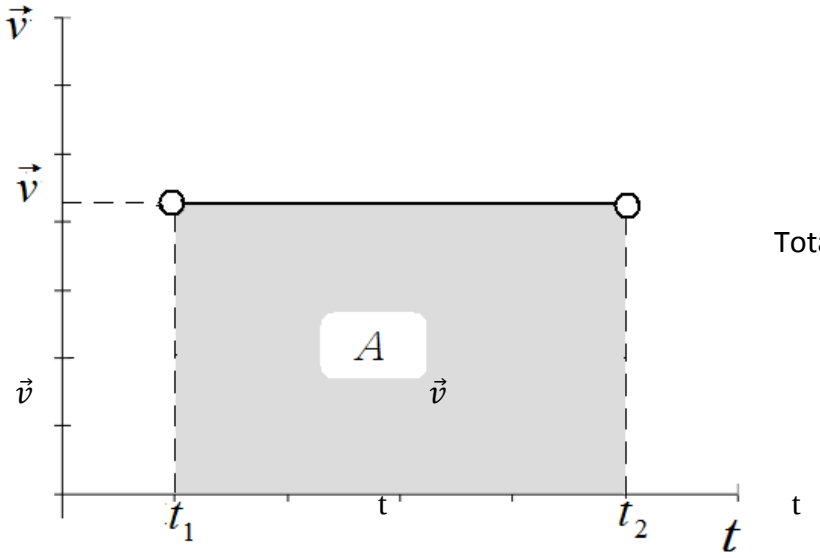
speeding up, negative direction



slowing down, positive direction

<u>Displacement from a Velocity vs Time Graph</u>

What if we were to calculate the area under the 'curve' on a velocity vs time graph?

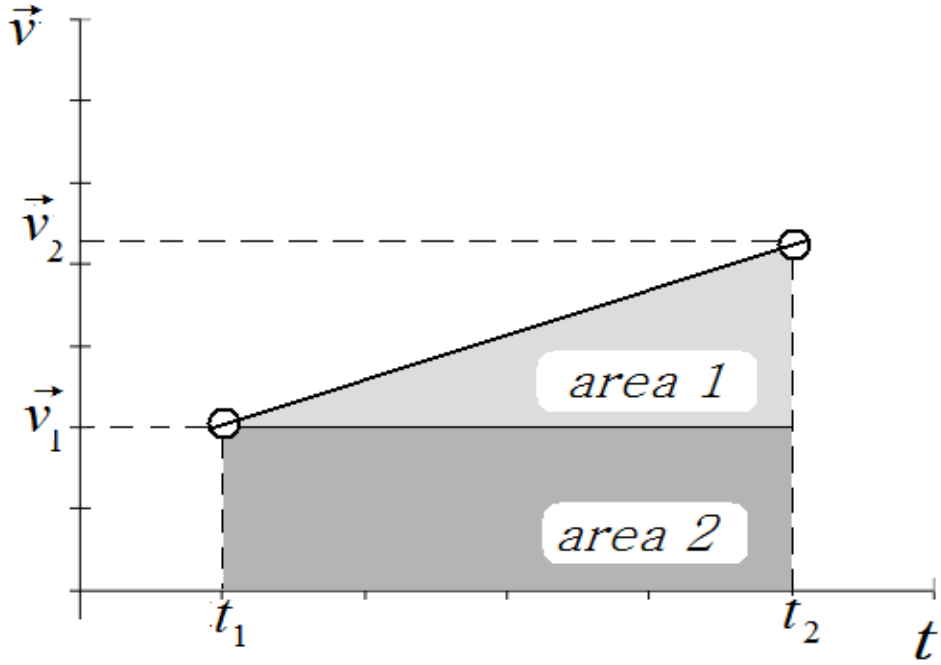


Total area (*A*) = *length x width*

$$A = l \cdot w$$
$$A = (t_2 - t_1)(\vec{v} - 0)$$
$$A = \vec{v}\Delta t$$
$$A = \Delta \vec{d}$$

Therefore, the area under the 'curve; of a velocity vs time graph is displacement!!!

Or, in general:



Total area (*A*) = *area 1* + *area 2*

$$A = \left(\frac{1}{2}b \cdot h\right) + (l \cdot w)$$
$$A = \left[\frac{1}{2}(t_2 - t_1)(\vec{v}_2 - \vec{v}_1)\right] + [(t_2 - t_1)(\vec{v}_1 - 0)]$$
$$A = \frac{1}{2}\vec{v}_2\Delta t - \frac{1}{2}\vec{v}_1\Delta t + \vec{v}_1\Delta t$$
$$A = \frac{1}{2}\vec{v}_2\Delta t + \frac{1}{2}\vec{v}_1\Delta t$$
$$A = \frac{1}{2}(\vec{v}_2 + \vec{v}_1)\Delta t$$
$$A = \vec{v}_{avg}\Delta t$$
$$A = \Delta\vec{d}$$

*To remember:

- position vs time graphs will always be smooth in reality, and can contain curved lines.
- velocity vs time graphs (for uniform accelerated motion) will consist of straight lines that can change abruptly