Unit I – Kinematics

Topics covered:

- Distance, Displacement, Position
- Speed, Velocity
- Acceleration
- 5 Kinematic Equations
- 1-D Projectile Motion
- 2-D Vectors
- 2-D Projectile Motion

Distance, Displacement, Position

Kinematics: The study of *how* objects move

Scalar: a quantity with a magnitude but no direction e.g. I walked 1.6 km to the store
Some values can only be scalars: e.g. temperature (°C), mass (kg)

Vector: a quantity with both magnitude and direction
e.g. I walked 1.6 km [E] to the store
*symbols with arrows on top of them are vectors – arrows imply directions! (e.g. $\vec{d}$)
symbols without arrows are scalars (e.g. d)

Origin: An arbitrary reference point from which all measurements are made.
*An object does not have to begin its motion at the origin!

Δ = **delta**: In science, *delta* most often means change. Any measured change is a comparison between a final state and an initial state.

OR: $\Delta x = x_f - x_i$

e.g. it was 6°C, and now it is -2°C. The change in temperature was -8°C, or 8°C colder

=> $\Delta T = T_f - T_i = -2°C - (6°C)$
$= -8°C$

Position ($\vec{d}$)

A vector measurement of an object's location with reference to a chosen point (the origin).
e.g. My sister's house is 47 km [N] of Toronto

Distance (d)

A scalar measurement of the total length of a journey

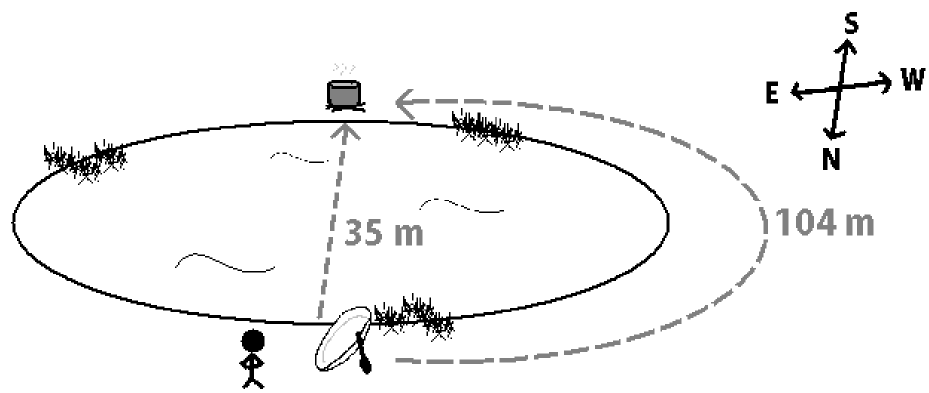
e.g. We drove 56 km to my sister's house

Displacement ($\Delta\vec{d}$))

A vector value which shows how much an object's position has changed overall.

e.g. We travelled 47 km [N] to my sister's house

Ex #1: Consider the boat as the origin. The human walks around the pond to the pot.



Human's initial position:

$\vec{d}_i = 0\ m\ [S]$ (at origin)

Human's final position:

$\vec{d}_f = 35\ m\ [S]$

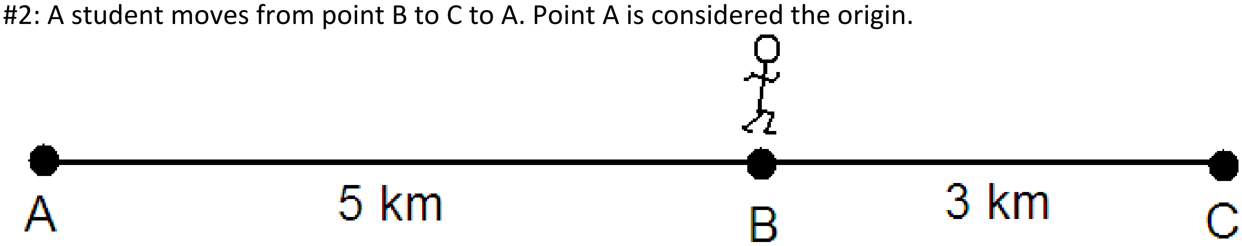
Total distance of journey:

$d = 104\ m$

Human's total displacement:

$$\begin{aligned}\Delta\vec{d} &= \vec{d}_f - \vec{d}_i \\ &= 35\ m\ [S] - 0\ m\ [S] \\ &= 35\ m\ [S]\end{aligned}$$

Ex #2: A student moves from point B to C to A. Point A is considered the origin.



Student's initial position: $\vec{d}_i = 5\ km\ [E]$

Student's final position: $\vec{d}_f = 0\ km\ [E]$

Total distance of journey: $d = 11\ km$

Student's total displacement:

$$\begin{aligned}\Delta\vec{d} &= \vec{d}_f - \vec{d}_i \\ &= 0\ km\ [E] - 5\ km\ [E] \\ &= -5\ km\ [E] \\ &= 5\ km\ [W]\end{aligned}$$

Speed, Velocity

Total Time *(t)*

the total time of a given event (e.g. it took 5 minutes to cook an egg)

Time Interval *(Δt)*

the time interval between two events

(e.g. 5 minutes passed between the time when the egg was placed in water and the time when it was fully cooked)

** often total time (t) and time interval (Δt) are the same*

average Speed *(v)*

a scalar quantity which shows the total distance of travel divided by the total time of travel

(e.g. we drove 50 km in 2 hours. Our speed was 25 km/h)

OR: $v = \frac{d}{t}$ units: m/s, km/h

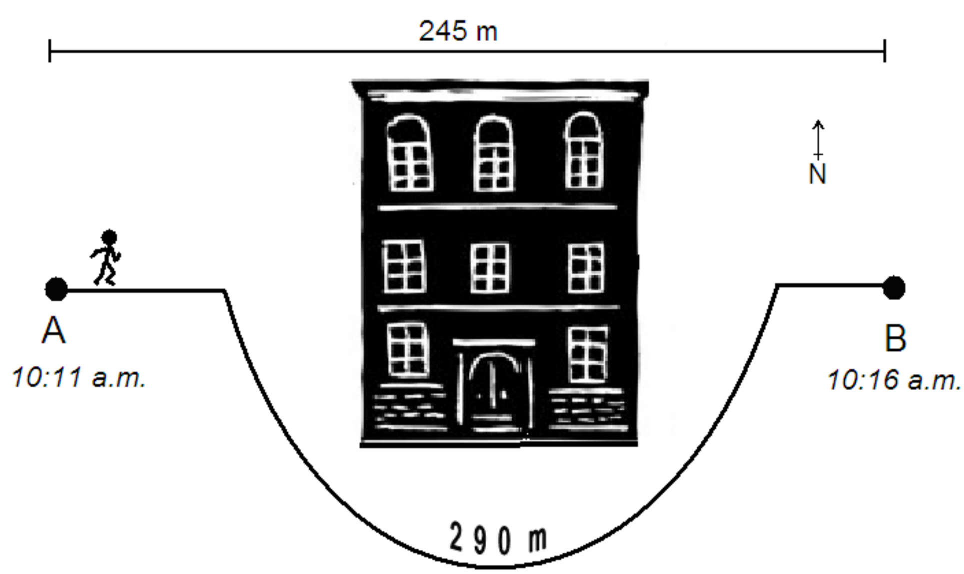
average Velocity *($\vec{v}$)*

a vector which shows the change in position divided by the time interval of that change.

(e.g. We ended up 40 km [N] from where we started after 4 hours. Our velocity was 10 km/h [N])

OR: $\vec{v} = \frac{\Delta\vec{d}}{\Delta t}$ *OR:* $\vec{v} = \frac{\vec{d}_f - \vec{d}_i}{t_f - t_i}$ units: m/s [?], km/h [?]

Ex #3: A child goes from point A to point B along the indicated path.



initial time: $t_i = 10{:}11$ a.m.

final time: $t_f = 10{:}16$ a.m.

Child's initial position: $\vec{d}_i = 0\ m\ [E]$

Child's final position: $\vec{d}_f = 245\ m\ [E]$

Total distance of journey: $d = 290\ m$

time interval: $\Delta t = t_f - t_i$

$= 10{:}16 \text{ a.m.} - 10{:}11 \text{ a.m.}$

$= 5.0 \text{ min}$

$= 3.0 \text{ x}10^2 \text{ s}$

Child's total displacement: $\Delta\vec{d} = \vec{d}_f - \vec{d}_i$

$= 245\, m\, [E] - 0\, m\, [E]$

$= 245\, m\, [E]$

Child's average speed: $v = \frac{d}{t}$

$= \frac{290\, m}{3.0\times10^2\, s}$

$= 0.97\, m/s$

Child's average velocity: $\vec{v} = \frac{\Delta\vec{d}}{\Delta t}$

$= \frac{245\, m\, [E]}{3.0\times10^2\, s}$

$= 0.83\, m/s\, [E]$

<u>**Non-Uniform Motion (acceleration!)**</u>

Uniform motion: motion of an object when its velocity is constant (could be 0 – at rest)

Non-Uniform motion: motion of an object when its velocity changes (undergoes acceleration)

Acceleration ($\vec{a}$)

a vector which shows the change in velocity divided by the time interval of that change.

(e.g. The bicycle got 0.5 m/s faster every second while moving forward. Its acceleration was $0.5 m/s^2$ [fwd])

OR: $\vec{a} = \frac{\Delta\vec{v}}{\Delta t}$ *OR:* $\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i}$ units: m/s^2 [?]

- Since velocity is a vector, it can change in two ways: in magnitude or direction. Acceleration occurs when at least one of these changes occur.

4 types of acceleration (2 positive, 2 negative):

speeding up (+) in the positive direction (+) = positive acceleration (+)

speeding up (+) in the negative direction (-) = negative acceleration (-)

slowing down (-) in the positive direction (+) = negative acceleration (-)

slowing down (-) in the negative direction (-) = positive acceleration (+)

Ex. #4 a ball rolls up a hill, then rolls back down. The ball started moving at 3.0 m/s [up the hill], then 4.5 s later, was moving at 4.0 m/s [down the hill].

$\vec{v}_1 = 3.0\ m/s\ [up\ the\ hill]$

$\vec{v}_2 = 4.0\ m/s\ [down\ the\ hill]$

$\Delta t = 4.5\ s$

$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$$

$$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

$$= \frac{4.0\ m/s\ [down] - \ 3.0\ m/s\ [up]}{4.5\ s}$$

$$= \frac{4.0\ m/s\ [down] - (-\ 3.0\ m/s\ [down])}{4.5\ s}$$

$$= \frac{7.0\ m/s\ [down]}{4.5\ s}$$

$$= 1.55555\ m/s^2\ [down]$$

$$= 1.6\ m/s^2\ [down]$$

<u>The Five (5) Kinematic Equations</u>

$$\vec{v}_2 = \vec{v}_1 + \vec{a}\Delta t \qquad \text{or} \qquad \vec{a} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t}$$

$$\Delta\vec{d} = \tfrac{1}{2}(\vec{v}_1 + \vec{v}_2)\Delta t \qquad \text{or} \qquad \Delta\vec{d} = \vec{v}_{avg}\Delta t$$

$$\Delta\vec{d} = \vec{v}_1\Delta t + \tfrac{1}{2}\vec{a}\Delta t^2$$

$$\Delta\vec{d} = \vec{v}_2\Delta t - \tfrac{1}{2}\vec{a}\Delta t^2$$

$$v_2^2 = v_1^2 + 2a\Delta d$$

- In Kinematics, there exist 5 variables of importance: $\Delta\vec{d},\ \vec{v}_2,\ \vec{v}_1, \vec{a}, \Delta t$
- There also exists 5 kinematic equations, each of which containing 4 variables (above)
- In any given question, you will have access to 3 variables, and will need to solve for the 4th
- Once you know 4 variables, you can use any other equation to find the 5th

<u>1-D Projectile Motion (Objects in Free Fall)</u>

A projectile is any object whose change in motion (acceleration) is caused only by the gravitational pull by a very large object (like the earth!). A thrown pie, once it has left your hand, is a perfect example of a projectile (except for the waste of food).

Galileo Galilei, over 350 years ago, was really interested in objects in freefall, and found out two very important things (if all friction, including air resistance, is ignored):

1. The rate at which ANY object accelerates due to gravity is INDEPENDENT of its mass (n.b. Mass does not appear in any of the 5kinematic equations for uniform linear acceleration above).

2. The rate at which ANY object accelerates during free fall (near the surface of the earth) is CONSTANT: $\vec{a} = 9.8\ m/s^2 [down]$

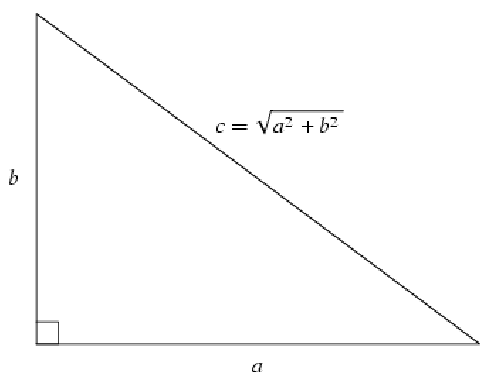
Facts to remember:

- Acceleration due to gravity is constant throughout the object's motion in magnitude and direction (9.8 $^{m}/_{s^2}$ [down] 'near' the surface of the earth)
- At the maximum height of the object's motion, its velocity = 0 m/s
- Δt to reach max height = Δt to return from max height to original position

2-D perpendicular vectors

To add vectors at right angles to each other algebraically, you will need to use:

Pythagoras' Theorem $a^2 + b^2 = c^2$



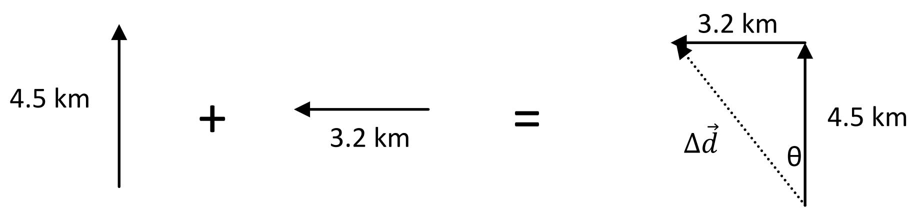
SOH CAH TOA

$$\sin\theta = \frac{opposite}{hypotenuse}$$

$$\cos\theta = \frac{adjacent}{hypotenuse}$$

$$\tan\theta = \frac{opposite}{adjacent}$$

e.g. A human moves 4.5 km [N] and then 3.2 km [W]. Find the total displacement of the human



$\Delta d = [(4.5\ km)^2 + (3.2\ km)^2]^{½}$

$\Delta d = 5.5\ km$

$\Delta\vec{d} = 5.5\ km\ [N\ 35\,°\ W]$

$\tan\theta = \frac{3.2\ km}{4.5\ km}$

$\theta = \tan^{-1}\left(\frac{3.2\ km}{4.5\ km}\right)$

$\theta = 35\,°$

2-D Projectile Motion (Objects in Free Fall)

To solve 2-D projectile motion problems, classify all motion variables as either occurring in the vertical (*y*) direction or the horizontal (*x*) direction.

Facts to remember:

- All motion in the vertical direction has no influence on the motion in the horizontal direction (and vice versa), so the two directions can be analyzed independently
- Gravity is the only force that acts on a projectile, so it will always accelerate in the vertical direction at the same rate (9.8 $m/s^2[down]$)
- As there are no forces acting the horizontal direction, the motion in the *x*-direction is uniform ($\vec{a} = 0$)
- The time it takes for the motion to occur in the *x*-direction is the same as that in the *y*-direction

e.g. A ball is thrown out of a building from a 2.5 m high window with an initial horizontal velocity of 3.2 m/s [E]. How far from the base of the building will the ball hit the ground, and what will be its final velocity?

x-direction	*y*-direction
Δt = ?	Δt = ?
$\vec{a}_x = 0$	$\vec{a}_y = 9.8\frac{m}{s^2}[down]$
$\Delta\vec{d}_x = ?$	$\Delta\vec{d}_y = 2.5\, m\ [down]$
$\vec{v}_x = 3.2\, m/s\ [E]$	$\vec{v}_{1_y} = 0\, m/s\ [down]$

From the *y*-direction, we can solve for the time, Δt, that the ball was in the air for:

$$\Delta\vec{d} = \vec{v}_1\Delta t + \tfrac{1}{2}\vec{a}\Delta t^2$$

$$\Delta\vec{d} = \tfrac{1}{2}\vec{a}\Delta t^2 \qquad \text{since } \vec{v}_{1_y} = 0$$

$$\Delta t = \left(\frac{2\vec{d}}{\vec{a}}\right)^{1/2}$$

$$\Delta t = \left(\frac{2(2.5\, m\ [down])}{9.8\, m/s^2[down]}\right)^{1/2}$$

$$\Delta t = 0.7143\ s$$

From the x-direction, we can now solve for how far from the base of the building will the ball hit the ground (i.e. its *range*) since we know that the time of flight in the vertical direction is always the same for that in the horizontal direction:

$$\Delta\vec{d} = \vec{v}_1\Delta t + \tfrac{1}{2}\vec{a}\Delta t^2$$
$$\Delta\vec{d} = \vec{v}\Delta t \qquad \text{since } \vec{a} = 0$$
$$\Delta\vec{d} = (3.2\ m/s\ [E])(0.7143\ s)$$
$$\Delta\vec{d} = 2.3\ m\ [E]$$

To determine the ball's final velocity, we need to know that any 2-D vector can be determined from its *x* and y components:

Since know there is no acceleration in the horizontal direction, the final velocity in the *x*-direction will be the same as it was initially:

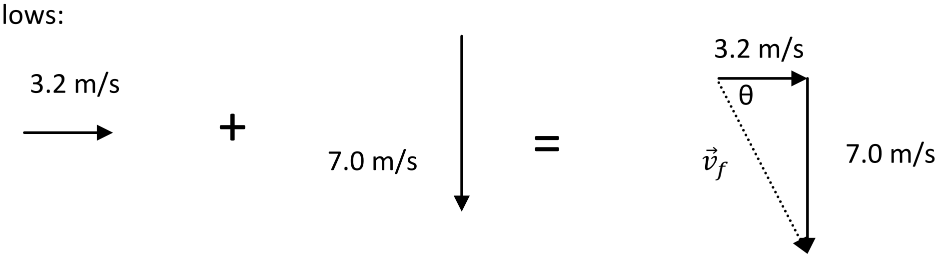
$$\vec{v}_{2_x} = \vec{v}_{1_x} = 3.2\ m/s\ [E]$$

The final velocity in the vertical direction can be calculated from what we already know about the motion in this direction:

$$v_2^2 = v_1^2 + 2a\Delta d$$
$$v_2^2 = 2a\Delta d$$
$$v_2 = \sqrt{2(9.8\ m/s^2)(2.5\ m)}$$
$$v_2 = 7.0\ m/s$$

$$\vec{v}_{2_y} = 7.0\ m/s\ [down]$$

Now that we have both the *x* and *y* components of the final velocity, we can add them as follows:



$$\vec{v}_f = [(3.2\ m/s)^2 + (7.0\ m/s)^2]^{1/2}$$
$$= 7.6968\ m/s$$

$$\tan\theta = \frac{7.0\ m/s}{3.2\ m/s}$$
$$\theta = \tan^{-1}\left(\frac{7.0}{3.2}\right)$$
$$\theta = 65.433\ °$$

$$\vec{v}_f = 7.7\ m/s\ [E\ 65\ °\ down]$$