Conceptual Questions

1. You have two springs that are identical except that spring ***1*** is stiffer than spring ***2***. On which spring is more work done (a) if they are stretched using the same force, (b) if they are stretched the same distance?[1]

   (*a*) In this case, the same force is applied to both springs. Spring 1 will stretch less, and so more work is done on spring 2.

   (*b*) In this case, both springs are stretched the same distance. It takes more force to stretch spring 1, and so more work is done on spring 1.

2. The drawing shows identical springs that are attached to a box in two different ways. Initially, the springs are unstrained. The box is then pulled to the right and released. In each case the initial displacement of the box is the same. At the moment of release, which box, if either, experiences the greater net force due to the springs? Provide a reason for your answer.[2]

   Same. In the first case both springs exert a pulling force to the left whose magnitude depends solely on ***k*** and ***x***. In the second case, even though one spring is a pushing and other one is pulling, the direction of both forces is to the left, and their magnitudes also depend solely on ***k*** and ***x***, which are constant in both cases.

3. A coil spring of mass ***m*** rests upright on a table. If you compress the spring by pressing down with your hand and then release it, can the spring leave the table? Explain.[3]

   The spring can leave the table if it is compressed enough. If the spring is compressed an amount x_0, then the gain in elastic PE is $\frac{1}{2}kx_0^2$. As the spring is compressed, its center of mass is lowered by some amount. If the spring is uniform, then the center of mass is lowered by $x_0/2$, and the amount of decrease in gravitational PE is $\frac{1}{2}mgx_0$. If the gain in elastic PE is more than the loss in gravitational PE, so that $\frac{1}{2}kx_0^2 > \frac{1}{2}mgx_0$ or $x_0 > mg/k$, then the released spring should rise up off of the table, because there is more than enough elastic PE to restore the spring to its original position. That extra elastic energy will enable the spring to "jump" off the table – it can raise its center of mass to a higher point and thus rise up off the table. Where does that "extra" energy come from? From the work you did in compressing the spring.

4. An object hangs motionless from a spring. When the object is pulled down, the sum of the elastic potential energy of the spring and the gravitational potential energy of the object and Earth.[4]

   a) increases b) stays the same c) decreases

   ANS: A

   Since a force was required to move the object downwards, work was done on it, which was then stored as elastic potential energy.

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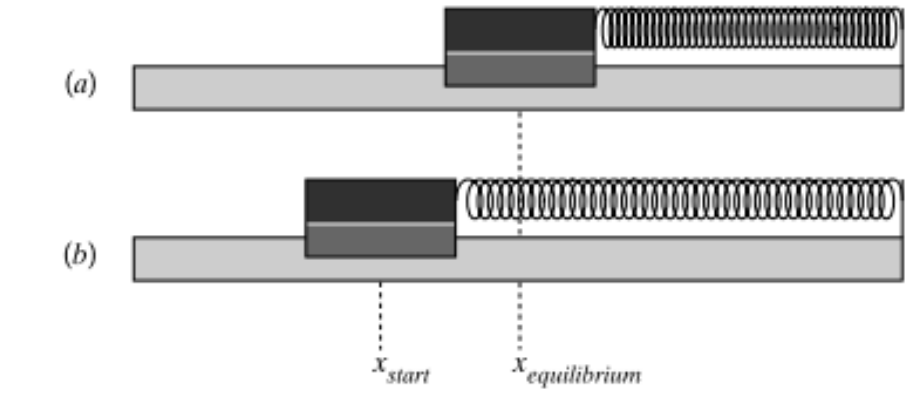
[1] Physics 6th Edition, Giancoli, Chapter 6 Questions, #7

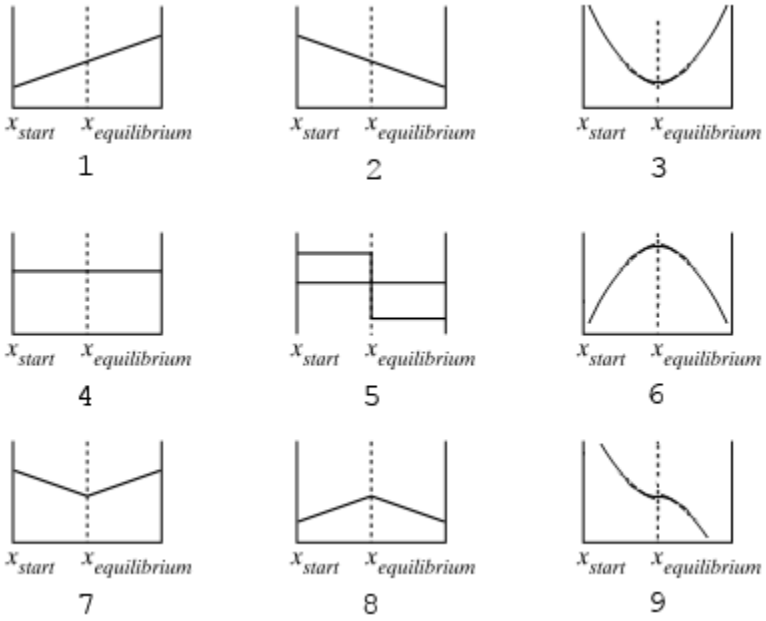
[2] Physics, 7th Edition, Cutnell & Johnson, Chapter 10 Conceptual Questions, #2

[3] Physics 6th Edition, Giancoli, Chapter 6 Questions, #12

[4] Peer Instruction – A User's Guide, Mazur, Oscillations CT 6

5. In part (a) of the figure, an air track cart attached to a spring rests on the track at the position $x_{equilibrium}$ and the spring is relaxed. In (b),the cart is pulled to the position x_{start} and released. It then oscillates about $x_{equilibrium}$. Which graph correctly represents the potential energy of the spring as a function of the position of the cart?[5]



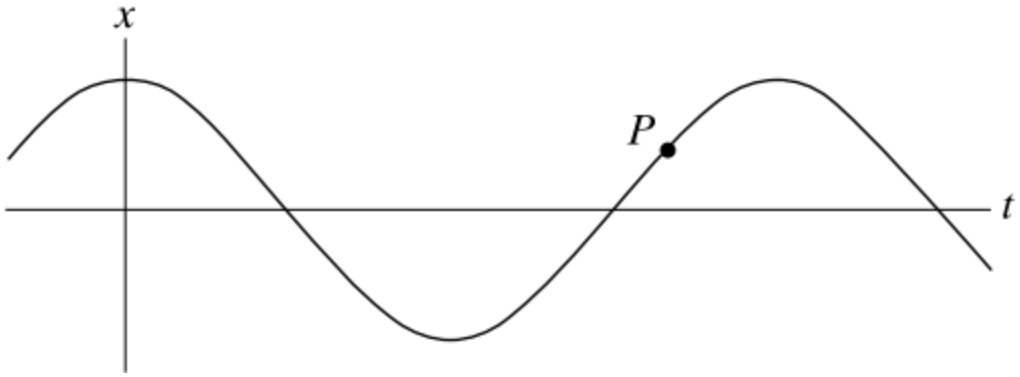


ANS: 3

Since elastic potential energy depends on the square of position, ***x***, the curve must be quadratic, which eliminate options 1,2,4,5,7,8. It also implies that as the magnitude of ***x*** increases, the potential energy will increase regardless of stretching or compressing. Also, it implies that at equilibrium position, elastic potential energy is a minimum. These point eliminate options 6,9.

[5] Peer Instruction – A User's Guide, Mazur, Interactions CT 7 (modified)

6. A mass attached to a spring oscillates back and forth as indicated in the position vs. time plot below. At point ***P***, the mass has [6]



a) positive velocity and positive acceleration.
b) positive velocity and negative acceleration.
c) positive velocity and zero acceleration.
d) negative velocity and positive acceleration.
e) negative velocity and negative acceleration.
f) negative velocity and zero acceleration.
g) zero velocity but is accelerating (positively or negatively).
h) zero velocity and zero acceleration.

ANS: B

The mass is moving in the positive direction and away from equilibrium (so positive velocity), but is also slowing down because the spring is pulling backwards on it (negative acceleration).

7. A particle is oscillating in simple harmonic motion. The time required for the particle to travel through one complete cycle is equal to the period of the motion, no matter what the amplitude is. But how can this be, since larger amplitudes mean that the particle travels farther? Explain.[7]

REASONING AND SOLUTION The time required for a particle in simple harmonic motion to travel through one complete cycle (the period) is independent of the amplitude of the motion, even though at larger amplitudes the particle travels further. This is possible because, at larger amplitudes, the maximum speed of the particle is greater. Thus, even though the particle must cover larger distances at larger amplitudes, it does so with greater speeds.

[6] Peer Instruction – A User's Guide, Mazur, Oscillations CT 4
[7] Physics, 7th Edition, Cutnell & Johnson, Chapter 10 Conceptual Questions, #7

Problems

8. A small truck is equipped with a rear bumper that has a spring constant of 5x10^7 N/m. The bumper can be compressed up to 15 cm without causing damage to the truck. What is the maximum velocity with which a solid 1000-kg car can collide with the bumper without causing damage to the truck?[8]

$W_{NC} = \Delta E_K + \Delta E_P$

$0 = E_{K2} - E_{K1} + E_{E2} - E_{E1}$

$0 = 0 - E_{K1} + E_{E2} - 0$

$E_{K1} = E_{E2}$

$\frac{1}{2}mv^2 = \frac{1}{2}kx^2$

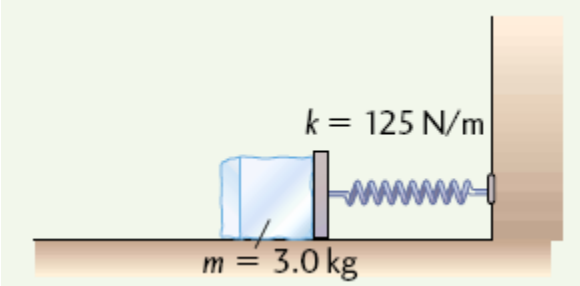
$v = \sqrt{\frac{k}{m}}x$

$v = \sqrt{\frac{8\times10^5\ N/m}{1\times10^3\ kg}}(0.15\ m)$

$v = 34\ m/s$

The car can collide with a speed of 34 m/s or less.

9. The diagram shows a 3.0-kg block of ice held against a spring with a force constant of 125 N/m. The spring is compressed by 12 cm.[9]



a) Calculate the speed of the ice just as it leaves the spring.

$W_{NC} = \Delta E_K + \Delta E_P$

$0 = E_{K2} - E_{K1} + E_{E2} - E_{E1}$

$0 = E_{K2} - 0 + 0 - E_{E1}$

$E_{K2} = E_{E1}$

$\frac{1}{2}mv^2 = \frac{1}{2}kx^2$

$v = \sqrt{\frac{k}{m}}x$

$v = \sqrt{\frac{125\ N/m}{3.0\ kg}}(0.12\ m)$

$v = 0.77\ m/s$

b) Calculate the speed of the ice just as it leaves the spring if the coefficient of friction between the plank and the ice is 0.10 (and assuming the spring is prevented from travelling past its equilibrium position).

$W_{NC} = \Delta E_K + \Delta E_P$

$F_f d\cos\theta = E_{K2} - E_{K1} + E_{E2} - E_{E1}$

$F_f d\cos\theta = E_{K2} - 0 + 0 - E_{E1}$

$E_{K2} = E_{E1} + F_f d\cos\theta$

$\frac{1}{2}mv^2 = \frac{1}{2}kx^2 + (-\mu mgd)$

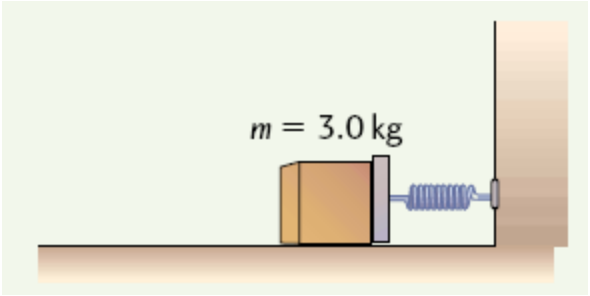
$v = \sqrt{\frac{k}{m}x^2 - gd}$

$v = \sqrt{\frac{125\ N/m}{3.0\ kg}(0.12\ m)^2 - 2(0.10)(9.8\ m/s^2)(0.12\ m)}$

$v = 0.60\ m/s$

[8] Physics Book Two, Irwin Publishing, Chapter 5 Problems, #45
[9] Physics Book Two, Irwin Publishing, Chapter 5 Problems, #46 (modified)

10. A spring with a force constant of 350 N/m (see below) is compressed 12 cm by a 3.0-kg mass. How fast is the mass moving after only 10 cm of the spring has been released?[10]



$W_{NC} = \Delta E_K + \Delta E_P$

$0 = E_{K2} - E_{K1} + E_{E2} - E_{E1}$

$0 = E_{K2} - 0 + E_{E2} - E_{E1}$

$E_{K2} = E_{E1} - E_{E2}$

$\frac{1}{2}mv^2 = \frac{1}{2}k(x_1^2 - x_2^2)$

$v = \sqrt{\frac{k}{m}(x_1^2 - x_2^2)}$

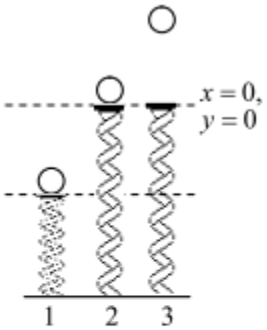
$v = \sqrt{\frac{350\ N/m}{3.0\ kg}((0.12\ m)^2 - (0.02\ m)^2)}$

$v = 1.28\ m/s$

11. A vertical spring (ignore mass), whose spring stiffness constant is 950 N/m, is attached to a table and is compressed down 0.150 m. [11]

a) What upward speed can it give a 0.30-kg ball when released?

Use conservation of energy. The level of the ball on the uncompressed spring taken as the zero location for both gravitational PE $(y = 0)$ and elastic PE $(x = 0)$. Take up to be positive for both.



(*a*) Subscript 1 represents the ball at the launch point, and subscript 2 represents the ball at the location where it just leaves the spring, at the uncompressed length. We have $v_1 = 0$, $x_1 = y_1 = -0.150\,\text{m}$, and $x_2 = y_2 = 0$. Solve for v_2.

$$E_1 = E_2 \rightarrow \tfrac{1}{2}mv_1^2 + mgy_1 + \tfrac{1}{2}kx_1^2 = \tfrac{1}{2}mv_2^2 + mgy_2 + \tfrac{1}{2}kx_2^2 \rightarrow$$

$$0 + mgy_1 + \tfrac{1}{2}kx_1^2 = \tfrac{1}{2}mv_2^2 + 0 + 0 \rightarrow v_2 = \sqrt{\frac{kx_1^2 + 2mgy_1}{m}}$$

$$v_2 = \sqrt{\frac{(950\,\text{N/m})(0.150\,\text{m})^2 + 2(0.30\,\text{kg})(9.80\,\text{m/s}^2)(-0.150\,\text{m})}{(0.30\,\text{kg})}} = \boxed{8.3\,\text{m/s}}$$

b) How high above its original position (spring compressed) will the ball fly?

(*b*) Subscript 3 represents the ball at its highest point. We have $v_1 = 0$, $x_1 = y_1 = -0.150\,\text{m}$, $v_3 = 0$, and $x_3 = 0$. Solve for y_3.

$$E_1 = E_3 \rightarrow \tfrac{1}{2}mv_1^2 + mgy_1 + \tfrac{1}{2}kx_1^2 = \tfrac{1}{2}mv_3^2 + mgy_3 + \tfrac{1}{2}kx_3^2 \rightarrow$$

$$0 + mgy_1 + \tfrac{1}{2}kx_1^2 = 0 + mgy_2 + 0 \rightarrow y_2 - y_1 = \frac{kx_1^2}{2mg} = \frac{(950\,\text{N/m})(0.150\,\text{m})^2}{2(0.30\,\text{kg})(9.80\,\text{m/s}^2)} = \boxed{3.6\,\text{m}}$$

[10] Physics Book Two, Irwin Publishing, Chapter 5 Problems, #47
[11] Physics 6th Edition, Giancoli, Chapter 6 Problems, #39

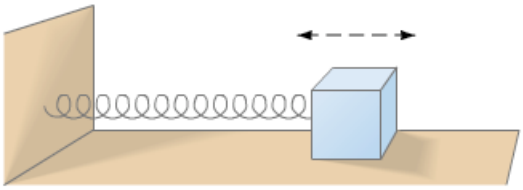
12. A 0.620-kg wood block is firmly attached to a very light horizontal spring (k = 180 N/m, see below). It is noted that the block-spring system, when compressed 5.0 cm and released, stretches out 2.3 cm beyond the equilibrium position before stopping and turning back. What is the coefficient of kinetic friction between the block and the table? [12]

Use conservation of energy, including the non-conservative frictional force. The block is on a level surface, so there is no gravitational PE change to consider. The frictional force is given by $F_{fr} = \mu_k F_N = \mu_k mg$, since the normal force is equal to the weight. Subscript 1 represents the block at the compressed location, and subscript 2 represents the block at the maximum stretched position. The location of the block when the spring is neither stretched nor compressed is the zero location for elastic PE ($x = 0$). Take right to be the positive direction. We have $v_1 = 0$, $x_1 = -0.050\text{ m}$, $v_2 = 0$, and $x_2 = 0.023\text{ m}$.

$$W_{NC} + E_1 = E_2 \rightarrow F_{fr}\Delta x\cos 180^\circ + \tfrac{1}{2}mv_1^2 + \tfrac{1}{2}kx_1^2 = \tfrac{1}{2}mv_2^2 + \tfrac{1}{2}kx_2^2 \rightarrow$$

$$-\mu_k mg\Delta x + \tfrac{1}{2}kx_1^2 = \tfrac{1}{2}kx_2^2 \rightarrow$$

$$\mu_k = \frac{k\left(x_1^2 - x_2^2\right)}{2mg\Delta x} = \frac{(180\text{ N/m})\left[(0.050\text{m})^2 - (0.023\text{m})^2\right]}{2(0.620\text{ kg})\left(9.80\text{ m/s}^2\right)(0.073\text{ m})} = \boxed{0.40}$$



13. A 280-g wood (see image above) can slide along a table where the coefficient of friction is 0.30. A force of 22 N compresses the spring 18 cm. If the spring is released from this position, how far beyond its equilibrium position will it stretch at its first maximum extension?[13]

Use conservation of energy, including the non-conservative frictional force. The block is on a level surface, so there is no gravitational PE change to consider. Since the normal force is equal to the weight, the frictional force is $F_{fr} = \mu_k F_N = \mu_k mg$. Subscript 1 represents the block at the compressed location, and subscript 2 represents the block at the maximum stretched position. The location of the block when the spring is neither stretched nor compressed is the zero location for elastic PE ($x = 0$). Take right to be the positive direction. We have $v_1 = 0$, $x_1 = -0.18\text{ m}$, and $v_2 = 0$. The value of the spring constant is found from the fact that a 20-N force compresses the spring 18 cm, and so $k = F/x = 22\text{ N}/0.18\text{ m} = 122.2\text{ N/m}$. The value of x_2 must be positive.

$$W_{NC} + E_1 = E_2 \rightarrow F_{fr}\Delta x\cos 180^\circ + \tfrac{1}{2}mv_1^2 + \tfrac{1}{2}kx_1^2 = \tfrac{1}{2}mv_2^2 + \tfrac{1}{2}kx_2^2 \rightarrow$$

$$-\mu_k mg\left(x_2 - x_1\right) + \tfrac{1}{2}kx_1^2 = \tfrac{1}{2}kx_2^2 \rightarrow x_2^2 + \frac{2\mu_k mg}{k}x_2 - \left(\frac{2\mu_k mg}{k}x_1 + x_1^2\right) = 0 \rightarrow$$

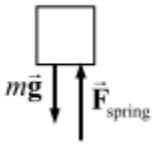
$$x_2^2 + \frac{2(0.30)(0.28)(9.80)}{122.2}x_2 - \left(\frac{2(0.30)(0.28)(9.80)}{122.2}(-0.18) + (-0.18)^2\right) = 0 \rightarrow$$

$$x_2^2 + 0.01347x_2 - 0.02997 = 0 \rightarrow x_2 = 0.1665\text{ m}, -0.1800\text{m} \rightarrow x_2 = \boxed{0.17\text{ m}}$$

[12] Physics 6th Edition, Giancoli, Chapter 6 Problems, #55
[13] Physics 6th Edition, Giancoli, Chapter 6 Problems, #56

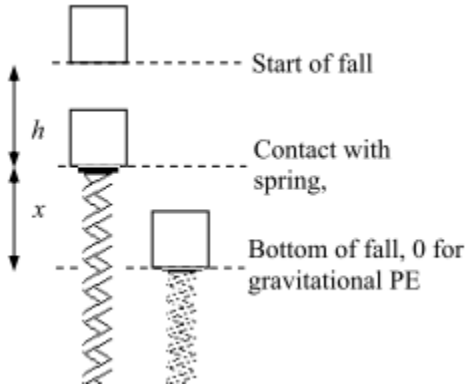
14. An engineer is designing a spring to be placed at the bottom of an elevator shaft. If the elevator cable should break when the elevator is at a height ***h*** above the top of the spring, calculate the value that the spring stiffness constant ***k*** should have so that passengers undergo an acceleration of no more than 5.0***g*** when brought to rest. Let ***M*** be the total mass of the elevator and passengers. [14]

The maximum acceleration of 5.0 g occurs where the force is at a maximum. The maximum force occurs at the bottom of the motion, where the spring is at its maximum compression. Write Newton's 2nd law for the elevator at the bottom of the motion, with up as the positive direction.



$$F_{net} = F_{spring} - Mg = Ma = 5.0Mg \quad \rightarrow \quad F_{spring} = 6.0Mg$$

Now consider the diagram for the elevator at various points in its motion. If there are no non-conservative forces, then mechanical energy is conserved. Subscript 1 represents the elevator at the start of its fall, and subscript 2 represents the elevator at the bottom of its fall. The bottom of the fall is the zero location for gravitational PE $(y = 0)$. There is also a point at the top of the spring that we will define as the zero location for elastic PE ($x = 0$). We have $v_1 = 0$, $y_1 = x + h$, $x_1 = 0$, $v_2 = 0$, $y_2 = 0$, and $x_2 = x$. Apply conservation of energy.



$$E_1 = E_2 \quad \rightarrow \quad \tfrac{1}{2}Mv_1^2 + Mgy_1 + \tfrac{1}{2}kx_1^2 = \tfrac{1}{2}Mv_2^2 + Mgy_2 + \tfrac{1}{2}kx_2^2 \quad \rightarrow$$

$$0 + Mg(x+h) + 0 = 0 + 0 + \tfrac{1}{2}kx_2^2 \quad \rightarrow \quad Mg(x+h) = \tfrac{1}{2}kx_2^2$$

$$F_{spring} = 6.0Mg = kx \quad \rightarrow \quad x = \frac{6.0Mg}{k} \quad \rightarrow \quad Mg\left(\frac{6Mg}{k} + h\right) = \tfrac{1}{2}k\left(\frac{6Mg}{k}\right)^2 \quad \rightarrow \quad \boxed{k = \frac{12Mg}{h}}$$

15. A mattress manufacturer estimates that 20 springs are required to comfortably support a 100-kg person. When supporting the person, the 20 springs are compressed 3.5 cm. Calculate the spring constant for one spring.[15]

$F = ma$

$F = (100\text{ kg})(9.8\text{ N/kg})$

$F = 980\text{ N}$

Divided into 20 springs:

$$F = \frac{980\text{ N}}{20}$$

$F = 49\text{ N per spring}$

$F = kx$

$49\text{ N} = k(0.035\text{ m})$

$k = 1.4 \times 10^3\text{ N/m}$

[14] Physics 6th Edition, Giancoli, Chapter 6 Problems, #45

[15] Physics Book Two, Irwin Publishing, Chapter 5 Problems, #49

16. A small ball is attached to one end of a spring that has an unstrained length of 0.200 m. The spring is held by the other end, and the ball is whirled around in a horizontal circle at a speed of 3.00 m/s. The spring remains nearly parallel to the ground during the motion and is observed to stretch by 0.010 m. By how much would the spring stretch if it were attached to the ceiling and the ball allowed to hang straight down, motionless?[16]

REASONING When the ball is whirled in a horizontal circle of radius r at speed v, the centripetal force is provided by the restoring force of the spring. From Equation 5.3, the magnitude of the centripetal force is mv^2/r, while the magnitude of the restoring force is kx (see Equation 10.2). Thus,

$$\frac{mv^2}{r} = kx \qquad (1)$$

The radius of the circle is equal to $(L_0 + \Delta L)$, where L_0 is the unstretched length of the spring and ΔL is the amount that the spring stretches. Equation (1) becomes

$$\frac{mv^2}{L_0 + \Delta L} = k\Delta L \qquad (1')$$

If the spring were attached to the ceiling and the ball were allowed to hang straight down, motionless, the net force must be zero: $mg - kx = 0$, where $-kx$ is the restoring force of the spring. If we let Δy be the displacement of the spring in the vertical direction, then

$$mg = k\Delta y$$

Solving for Δy, we obtain

$$\Delta y = \frac{mg}{k} \qquad (2)$$

SOLUTION According to equation (1') above, the spring constant k is given by

$$k = \frac{mv^2}{\Delta L(L_0 + \Delta L)}$$

Substituting this expression for k into equation (2) gives

$$\Delta y = \frac{mg\Delta L(L_0 + \Delta L)}{mv^2} = \frac{g\Delta L(L_0 + \Delta L)}{v^2}$$

or

$$\Delta y = \frac{(9.80\ \text{m/s}^2)(0.010\ \text{m})(0.200\ \text{m} + 0.010\ \text{m})}{(3.00\ \text{m/s})^2} = \boxed{2.29 \times 10^{-3}\ \text{m}}$$

[16] Physics, 7th Edition, Cutnell & Johnson, Chapter 10 Problems, #11

17. A 0.25-kg mass is attached to the end of a spring that is attached horizontally to a wall. When the mass is displaced 8.5 cm and then released, it undergoes SHM. The amplitude remains constant and k = $1.4x10^2$ N/m.[17]

a) How far does the mass move in the first 5 cycles?

$m = 0.25$ kg
$A = 8.5$ cm
$k = 1.4 \times 10^2$ N/m
(a) $d = ?$

During each cycle, the mass moves 4 amplitudes, or $4A$. In 5 cycles, the mass will move:

$$d = 5 \times 4A$$
$$= 5 \times 4(8.5 \text{ cm})$$
$$d = 1.7 \times 10^2 \text{ cm}$$

The mass moves 1.7×10^2 cm in the first five cycles.

b) What is the period of vibration of the mass-spring system?

(b) $T = ?$

$$T = 2\pi\sqrt{\frac{m}{k}}$$
$$= 2\pi\sqrt{\frac{0.25 \text{ kg}}{1.4 \times 10^2 \text{ N/m}}}$$
$$T = 0.27 \text{ s}$$

The period of vibration is 0.27 s.

18. Prove that the maximum speed of a mass on a spring in SHM is given by $\boldsymbol{2\pi fx}$.[18]

For the maximum speed, all of the elastic energy will be converted to kinetic:

$$E_{T1} = E_{T2}$$
$$\frac{1}{2}kx^2 = \frac{1}{2}mv^2$$
$$v = \sqrt{\frac{kx^2}{m}}$$
$$v = x\sqrt{\frac{k}{m}}$$

For a SHM system, $x = A$, therefore:

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$
$$2\pi f = \sqrt{\frac{k}{m}}$$

Substituting above:

$$v = x\sqrt{\frac{k}{m}}$$
$$= A(2\pi f)$$
$$v = 2\pi fA$$

[17] Physics 12, Nelson Education, Section 4.5 Practice, #18
[18] Physics 12, Nelson Education, Section 4.5 Practice, #26

19. A bungee jumper with mass 65.0 kg jumps from a high bridge. After reaching his lowest point, he oscillates up and down, hitting a low point eight more times in 38.0 s. He finally comes to rest 25.0 m below the level of the bridge. Calculate the spring constant and the unstretched length of the bungee cord. [19]

The period of the jumper's motion is $T = \dfrac{38.0\text{ s}}{8\text{ cycles}} = 4.75\text{ s}$. The spring constant can then be found from the period and the jumper's mass.

$$T = 2\pi\sqrt{\frac{m}{k}} \quad \rightarrow \quad k = \frac{4\pi^2 m}{T^2} = \frac{4\pi^2 (65.0\text{ kg})}{(4.75\text{ s})^2} = 113.73\text{ N/m} \approx \boxed{114\text{ N/m}}$$

The stretch of the bungee cord needs to provide a force equal to the weight of the jumper when he is at the equilibrium point.

$$k\Delta x = mg \quad \rightarrow \quad \Delta x = \frac{mg}{k} = \frac{(65.0\text{ kg})(9.80\text{ m/s}^2)}{113.73\text{ N/m}} = 5.60\text{ m}$$

Thus the unstretched bungee cord must be $25.0\text{ m} - 5.60\text{ m} = \boxed{19.4\text{ m}}$

[19] Physics 6th Edition, Giancoli, Chapter 11 Problems, #27