

### Conceptual Questions

1. Two satellites **A** and **B** of the same mass are going around Earth in concentric orbits. The distance of satellite **B** from Earth's center is twice that of satellite **A**. What is the ratio of the tangential speed of **B** to that of **A**?<sup>1</sup>

a)  $\frac{1}{2}$

b)  $\frac{1}{\sqrt{2}}$

c) 1

d)  $\sqrt{2}$

e) 2

ANS: B

Both satellites orbit the Earth due to their mutual gravitational force of attraction:

$$\frac{Gm_{\text{Earth}}m_{\text{satellite}}}{r^2} = \frac{v^2}{r}$$

$$m_{\text{satellite}} = \frac{v^2 r}{Gm_{\text{Earth}}}$$

Since both masses are equal

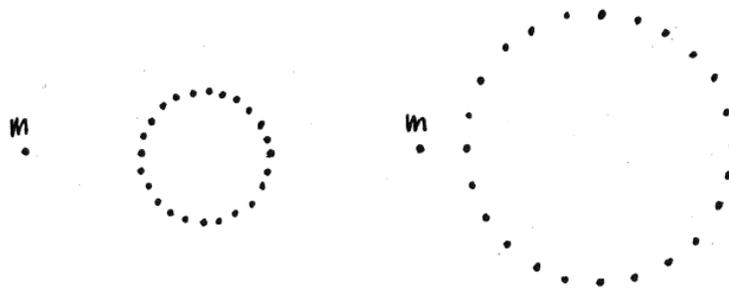
$$m_A = \frac{v_A^2 R}{Gm_{\text{Earth}}} = m_B = \frac{v_B^2 2R}{Gm_{\text{Earth}}}$$

$$v_B^2 = \frac{v_A^2 R}{2R}$$

$$v_B = \frac{v_A}{\sqrt{2}}$$

2. A little mass **m** is a certain distance from the centre of a globular cluster of masses and there is a certain force of gravity, due to the cluster of masses, on **m** that pulls it toward the centre of the cluster. If neither **m** or the centre of the cluster moves, but uniformly expands, the force of gravity on **m** from the cluster will<sup>2</sup>

- a) increase  
b) decrease  
c) remain unchanged



ANS: C

Newton's law of universal

gravitation assumes point masses. If the masses are sufficiently far away, the assumption is very appropriate. At smaller distances, an object's *centre of mass* is used when calculating, and for perfectly symmetric object (or groups of objects, like above), the centre of mass will be located at the centre of the object. Hence, neither the distance nor the direction from cluster's centre of mass changes in both cases above, so neither does the gravitational force (Another way of looking at it is as some parts of the cluster get closer, others get equally further and so both changes cancel each other out).

<sup>1</sup> Peer Instruction – A User's Guide, Mazur, Gravitation CT 3

<sup>2</sup> Peer Instruction – A User's Guide, Mazur

3. Choose **each** of the following locations that could be considered as a non-inertial frame of reference:<sup>3</sup>
- a) on a merry-go-round
  - b) on the equator
  - c) on the north or south pole
  - d) on the sun
  - e) at the centre of the milky way

ANS: A, B, C, D, E

A merry-go-round is spinning and so accelerating. The Earth spins on its axis and so every point on the equator (and all others not on the poles) is spinning. The Earth itself is rotating around the Sun, and so the poles are still accelerating. The Sun rotates around the centre of our spiral milky way galaxy, and our galaxy is also accelerating due the gravitational attraction of all the other galaxies around it.

4. The Sun's gravitational pull on the Earth is much larger than the Moon's, yet the moon is mainly responsible for the tides. Explain [Hint: Consider the difference in gravitational pull from one side of the Earth to the other.]<sup>4</sup>

The difference in force on the two sides of the Earth from the gravitational pull of either the Sun or the Moon is the primary cause of the tides. That difference in force comes about from the fact that the two sides of the Earth are a different distance away from the pulling body. Relative to the Sun, the difference in distance (Earth diameter) of the two sides from the Sun, relative to the average distance to the Sun, is given by  $2R_{\text{Earth}}/R_{\text{Earth to Sun}} = 8.5 \times 10^{-5}$ . The corresponding relationship between

the Earth and the Moon is  $2R_{\text{Earth}}/R_{\text{Earth to Moon}} = 3.3 \times 10^{-2}$ . Since the relative change in distance is much greater for the Earth-Moon combination, we see that the Moon is the primary cause of the Earth's tides.

5. Will an object weigh more at the equator or at the poles? [Hint: Consider the difference in motion of the object and the parts of the Earth at the equator compared to the poles.]<sup>5</sup>

An object weighs more at the poles, due to two effects which complement (not oppose) each other. First of all, the Earth is slightly flattened at the poles and expanded at the equator, relative to a perfect sphere. Thus the mass at the poles is slightly closer to the center, and so experiences a slightly larger gravitational force. Secondly, objects at the equator have a centripetal acceleration due to the rotation of the Earth that objects at the poles do not have. To provide that centripetal acceleration, the apparent weight (the radially outward normal force of the Earth on an object) is slightly less than the gravitational pull inward. So the two effects both make the weight of an object at the equator less than that at the poles.

<sup>3</sup> Almeida, F., Physics Department, Victoria Park C.I.

<sup>4</sup> Physics 6<sup>th</sup> Edition, Giancoli, Chapter 5 Questions, #14

<sup>5</sup> Physics 6<sup>th</sup> Edition, Giancoli, Chapter 5 Questions, #15

6. The mass of Pluto was not known until it was discovered to have a moon. Explain how this discovery enabled an estimation of Pluto's mass.<sup>6</sup>

Let the mass of Pluto be  $M$ , the mass of the moon be  $m$ , the radius of the moon's orbit be  $R$ , and the period of the moon's orbit be  $T$ . Then Newton's second law for the moon orbiting Pluto will be

$F = \frac{GmM}{R^2}$ . If that moon's orbit is a circle, then the form of the force must be centripetal, and so

$F = mv^2/R$ . Equate these two expressions for the force on the moon, and substitute the relationship for a circular orbit that  $v = 2\pi R/T$ .

$$\frac{GmM}{R^2} = \frac{mv^2}{R} = \frac{4\pi^2 mR}{T^2} \rightarrow M = \frac{4\pi^2 R^3}{GT^2}.$$

Thus a value for the mass of Pluto can be calculated knowing the period and radius of the moon's orbit.

### Problems

7. Earth is a satellite of the Sun with an orbit radius of  $1.5 \times 10^{11}$  m. What is the Sun's mass?<sup>7</sup>

$$G = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2,$$

$$T = 365 \text{ days} = 3.15 \times 10^7 \text{ s}$$

$$\frac{Gm_E m_S}{r^2} = m_E \left( \frac{4\pi^2 r}{T^2} \right)$$

$$m_S = \frac{4\pi^2 r^3}{GT^2}$$

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$$m_S = \frac{4\pi^2 (1.5 \times 10^{11} \text{ m})^3}{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(3.15 \times 10^7 \text{ s})^2}$$

$$m_S = 2.0 \times 10^{30} \text{ kg}$$

8. A satellite is in a circular orbit around an unknown planet. The satellite has a speed of  $1.70 \times 10^4$  m/s, and the radius of the orbit is  $5.25 \times 10^6$  m. A second satellite also has a circular orbit around this same planet. The orbit of this second satellite has a radius of  $8.60 \times 10^6$  m. What is the orbital speed of the second satellite?<sup>8</sup>

**REASONING** Equation 5.5 gives the orbital speed for a satellite in a circular orbit around the earth. It can be modified to determine the orbital speed around any planet **P** by replacing the mass of the earth  $M_E$  by the mass of the planet  $M_P$ :  $v = \sqrt{GM_P / r}$ .

**SOLUTION** The ratio of the orbital speeds is, therefore,

$$\frac{v_2}{v_1} = \frac{\sqrt{GM_P / r_2}}{\sqrt{GM_P / r_1}} = \sqrt{\frac{r_1}{r_2}}$$

Solving for  $v_2$  gives

$$v_2 = v_1 \sqrt{\frac{r_1}{r_2}} = (1.70 \times 10^4 \text{ m/s}) \sqrt{\frac{5.25 \times 10^6 \text{ m}}{8.60 \times 10^6 \text{ m}}} = \boxed{1.33 \times 10^4 \text{ m/s}}$$

<sup>6</sup> Physics 6<sup>th</sup> Edition, Giancoli, Chapter 5 Questions, #24

<sup>7</sup> Physics Book Two, Irwin Publishing, Chapter 2 Problems, #62a

<sup>8</sup> Physics, 7<sup>th</sup> Edition, Cutnell & Johnson, Chapter 5 Problems, #27

9. A satellite has a mass of 5850 kg and is in a circular orbit  $4.1 \times 10^5$  m above the surface of a planet. The period of the orbit is two hours. The radius of the planet is  $4.15 \times 10^6$  m. What is the true weight of the satellite when it is at rest on the planet's surface?<sup>9</sup>

**REASONING** The true weight of the satellite when it is at rest on the planet's surface can be found from Equation 4.4:  $W = (GM_p m) / r^2$  where  $M_p$  and  $m$  are the masses of the planet and the satellite, respectively, and  $r$  is the radius of the planet. However, before we can use Equation 4.4, we must determine the mass  $M_p$  of the planet.

The mass of the planet can be found by replacing  $M_E$  by  $M_p$  in Equation 5.6 and solving for  $M_p$ . When using Equation 5.6, we note that  $r$  corresponds to the radius of the circular orbit *relative to the center of the planet*.

**SOLUTION** The period of the satellite is  $T = 2.00 \text{ h} = 7.20 \times 10^3 \text{ s}$ . From Equation 5.6,

$$M_p = \frac{4\pi^2 r^3}{GT^2} = \frac{4\pi^2 [(4.15 \times 10^6 \text{ m}) + (4.1 \times 10^5 \text{ m})]^3}{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2 / \text{kg}^2)(7.20 \times 10^3 \text{ s})^2} = 1.08 \times 10^{24} \text{ kg}$$

Using Equation 4.4, we have

$$W = \frac{GM_p m}{r^2} = \frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2 / \text{kg}^2)(1.08 \times 10^{24} \text{ kg})(5850 \text{ kg})}{(4.15 \times 10^6 \text{ m})^2} = \boxed{2.45 \times 10^4 \text{ N}}$$

10. Four 9.5-kg spheres are located at the corners of a square of side 0.60 m. Calculate the magnitude and direction of the total gravitational force exerted on one of the spheres by the other three.<sup>10</sup>

Calculate the force on the sphere in the lower left corner, using the free-body diagram shown. From the symmetry of the problem, the net forces in the  $x$  and  $y$  directions will be the same. Note  $\theta = 45^\circ$

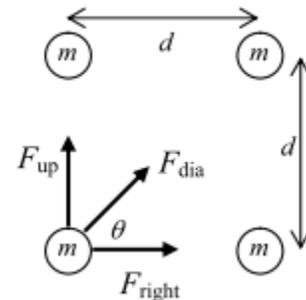
$$F_x = F_{\text{right}} + F_{\text{dia}} \cos \theta = G \frac{m^2}{d^2} + G \frac{m^2}{(\sqrt{2}d)^2} \frac{1}{\sqrt{2}} = G \frac{m^2}{d^2} \left( 1 + \frac{1}{2\sqrt{2}} \right)$$

and so  $F_y = F_x = G \frac{m^2}{d^2} \left( 1 + \frac{1}{2\sqrt{2}} \right)$ . The net force can be found by the

Pythagorean combination of the two component forces. Due to the symmetry of the arrangement, the net force will be along the diagonal of the square.

$$\begin{aligned} F &= \sqrt{F_x^2 + F_y^2} = \sqrt{2F_x^2} = F_x \sqrt{2} = G \frac{m^2}{d^2} \left( 1 + \frac{1}{2\sqrt{2}} \right) \sqrt{2} = G \frac{m^2}{d^2} \left( \sqrt{2} + \frac{1}{2} \right) \\ &= (6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2 / \text{kg}^2) \frac{(9.5 \text{ kg})^2}{(0.60 \text{ m})^2} \left( \sqrt{2} + \frac{1}{2} \right) = \boxed{3.2 \times 10^{-8} \text{ N at } 45^\circ} \end{aligned}$$

The force points towards the center of the square.



<sup>9</sup> Physics, 7<sup>th</sup> Edition, Cutnell & Johnson, Chapter 5 Problems, #33

<sup>10</sup> Physics 6<sup>th</sup> Edition, Giancoli, Chapter 5 Problems, #39

11. At what period must a cylindrical spaceship with a diameter of 32 m rotate with if occupants are to experience simulated gravity of  $0.60g$ ?<sup>11</sup>

The centripetal acceleration will simulate gravity. Thus  $v^2/r = 0.60g \rightarrow v = \sqrt{0.60gr}$ . Also for a rotating object, the speed is given by  $v = 2\pi r/T$ . Equate the two expressions for the speed and solve for the period.

$$v = \sqrt{0.60gr} = \frac{2\pi r}{T} \rightarrow T = \frac{2\pi r}{\sqrt{0.60gr}} = \frac{2\pi(16 \text{ m})}{\sqrt{(0.60)(9.8 \text{ m/s}^2)(16 \text{ m})}} = \boxed{10 \text{ sec}}$$

12. To create artificial gravity, the space station shown is rotating at a rate of 1.00 rpm. The radii of the cylindrically shaped chambers have the ratio  $\frac{r_A}{r_B} = 4.00$ . Each chamber **A** simulates an acceleration due to gravity of  $10.0 \text{ m/s}^2$ . Find the values for  $r_A$ ,  $r_B$  and the acceleration due to gravity simulated in chamber **B**.<sup>12</sup>

$$a_A = 4\pi^2 r_A f^2$$

$$r_A = \frac{a_A}{4\pi^2 f^2}$$

$$r_A = \frac{10.0 \text{ m/s}^2}{4\pi^2 \left( \frac{1.00 \text{ revolutions}}{\text{minute}} \cdot \frac{60 \text{ s}}{\text{minute}} \right)^2}$$

$$r_A = 911.89 \text{ m}$$

$$r_A = 912 \text{ m}$$

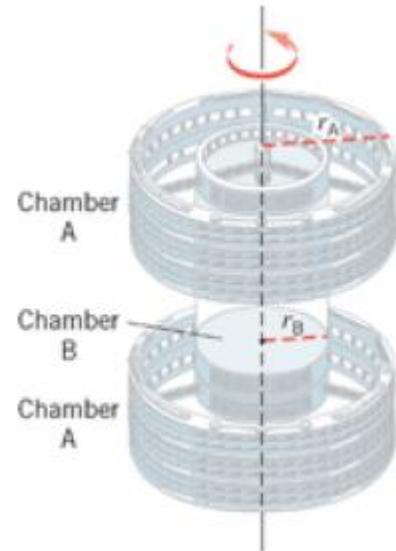
$$\frac{r_A}{r_B} = 4.00$$

$$r_B = \frac{r_A}{4.00}$$

$$r_B = \frac{912 \text{ m}}{4.00}$$

$$r_B = 227.97 \text{ m}$$

$$r_B = 228 \text{ m}$$



$$a_B = 4\pi^2 r_B f^2$$

$$a_B = 4\pi^2 (228 \text{ m}) \left( \frac{1.00 \text{ revolutions}}{\text{minute}} \cdot \frac{60 \text{ s}}{\text{minute}} \right)^2$$

$$a_B = 2.5003 \text{ m/s}^2$$

$$a_B = 2.50 \text{ m/s}^2$$

<sup>11</sup> Physics 6<sup>th</sup> Edition, Giancoli, Chapter 5 Problems, #45

<sup>12</sup> Physics, 7<sup>th</sup> Edition, Cutnell & Johnson, Chapter 5 Problems, #35

