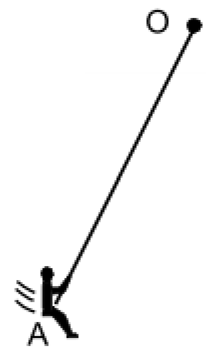
Conceptual Questions



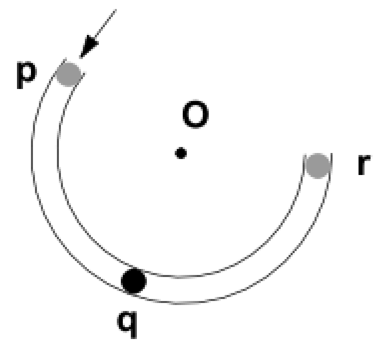
1. The figure below shows a boy swinging on a rope, starting at a point higher than A. Consider the following distinct forces:
   1. A downward force of gravity.
   2. A force exerted by the rope pointing from A to O.
   3. A force in the direction of the boy's motion.
   4. A force pointing from O to A.

Which of the above forces is (are) acting on the boy when he is at position A?[1]

(a) 1 only. (c) 1 and 3. (e) 1, 3, and 4.
(b) 1 and 2. (d) 1, 2, and 3.

ANS: B

Only the force of gravity and the tension force from the swing act on the person.



2. The accompanying figure shows a frictionless channel in the shape of a segment of a circle with center at "O". The channel has been anchored to a frictionless horizontal table top. You are looking down on the table. Forces exerted by the air are negligible. A ball is shot at high speed into the channel at "p" and exists at "r". Consider the following distinct forces:
   1. A downward force of gravity.
   2. A force exerted by the channel pointing from q to O.
   3. A force in the direction of motion.
   4. A force pointing from O to q.

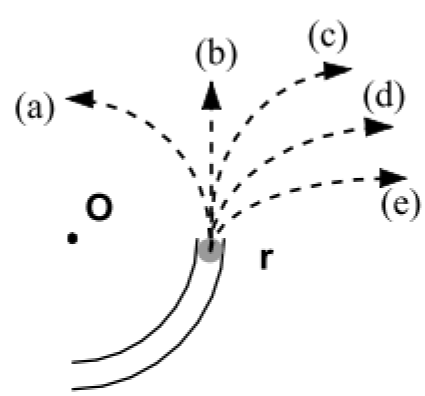
Which of the above forces is (are) acting on the ball when it is within the frictionless channel at position "q"?[2]

(a) 1 only. (c) 1 and 3. (e) 1, 3, and 4.
(b) 1 and 2. (d) 1, 2, and 3.

ANS: B

The force of gravity, the normal force from the table and the normal force from the channel are the only forces acting on the ball.

3. Which path in the figure on the right would the ball from question #2 most closely follow after it exits the channel at "r" and moves across the frictionless table top?[3]



ANS: B

The ball will travel tangentially to its circle of motion and since there is no friction between the ball and the table surface the ball will not spin in any other direction either.

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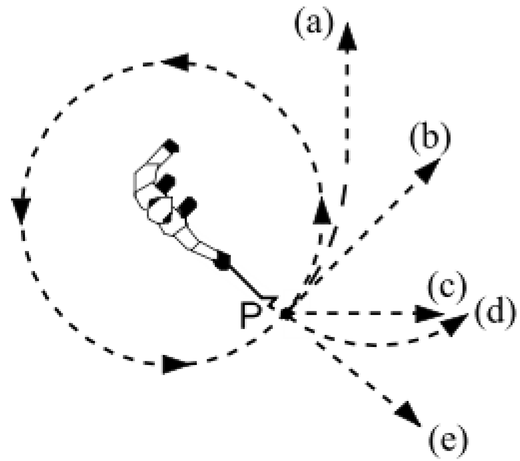
[1] Force Concept Inventory, Hestenes, Halloun, Wells, and Swackhamer, #18
[2] Force Concept Inventory, Hestenes, Halloun, Wells, and Swackhamer, #5
[3] Force Concept Inventory, Hestenes, Halloun, Wells, and Swackhamer, #6

4. A steel ball is attached to a string and is swung in a circular path in a horizontal plane as illustrated in the accompanying figure to the right. At the point P indicated in the figure, the string suddenly breaks near the ball. If these events are observed from directly above as in the figure, which path would the ball most closely follow after the string breaks? [4]



ANS: B

The ball will travel tangentially to its circle of motion

5. When a ball at rest hangs by a single vertical string, tension in the string is ***mg***. If the ball is made to move in a horizontal circle so that the string describes a cone, string tension
   (a) is ***mg***.
   (b) is greater than ***mg***, always.
   (c) is less than ***mg***, always.
   (d) may be greater or less than ***mg*** depending on the speed of the ball.[5]

ANS: B

The tension must prevent the ball from falling (canceling out the force of gravity) and accelerate it in a circular path, so the tension will always be greater.

6. What is the chance of a light car safely rounding an unbanked curve on an icy road as compared to that of a heavy car: worse, the same, or better? Assume that both cars have the same speed and are equipped with identical tires. Account for your answer.[6]

SAME. Solving for the speed required to safely negotiate the curve yields a relationship independent of mass (it cancels out). The extra friction force from the extra mass is negated by the larger centripetal force required to accelerate the car.

7. A stone is tied to a string and whirled around in a circle at a constant speed. Is the string more likely to break when the circle is horizontal or when it is vertical? Account for your answer, assuming the constant speed is the same in each case.[7]

Vertical. The magnitude of the tension force in both cases must counteract the force of gravity and supply the centripetal force. For the horizontal circle the forces are perpendicular and so their sum here is less than at the bottom of the vertical circle, where the forces are parallel. Therefore the string will most likely break at the bottom of the vertical circle.

8. Would a change in the earth's mass affect (a) the banking of airplanes as they turn, (b) the banking of roadbeds, (c) the speeds with which satellites are put into circular orbits, and (d) the performance of the loop-the-loop motorcycle stunt? In each case, give your reasoning.[8]

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[4] Force Concept Inventory, Hestenes, Halloun, Wells, and Swackhamer, #7
[5] 60 Questions – Basic Physics, Paul G, Hewitt, #14
[6] Physics, 7th Edition, Cutnell & Johnson, Chapter 5 Conceptual Questions, #8
[7] Physics, 7th Edition, Cutnell & Johnson, Chapter 5 Conceptual Questions, #14
[8] Physics, 7th Edition, Cutnell & Johnson, Chapter 5 Conceptual Questions, #13

a. Referring to Figure 5.10 in the text, we can see that the centripetal force on the plane is $L \sin\theta = mv^2/r$, where L is the magnitude of the lifting force. In addition, the vertical component of the lifting force must balance the weight of the plane, so that $L\cos\theta = mg$. Dividing these two equations reveals that $\tan\theta = v^2/(rg)$.

b. The banking condition for a car traveling at speed v around a curve of radius r, banked at angle θ is $\tan\theta = v^2/(rg)$, according to Equation 5.4 in the text.

c. The speed v of a satellite in a circular orbit of radius r about the earth is given by $v = \sqrt{GM_E/r}$, according to Equation 5.5 in the text.

d. The minimum speed required for a loop-the-loop trick around a loop of radius r is $v = \sqrt{rg}$, according to the discussion in Section 5.7 of the text. According to Equations 4.4 and 4.5, $g = GM_E/r^2$. Thus, any expression that depends on g also depends on M_E and would be affected by a change in the earth's mass. Such is the case for each of the four situations discussed above.

9. The last cycle in a washing machine is always the spin cycle, during which the drum rotates at high speed about a vertical axis. Explain how the spin cycle removes water from clothing.[9]

Inertia causes the water in your clothing to try to move in a straight line. If the drum in the washing machine were solid, it would apply a centripetal force on the water, which would keep it moving in a circle. Since the drum has holes in it, however, the water is able to leave the drum as it spins.

Problems

10. What is the maximum speed at which a 1500-kg car can round a curve on a flat road if the radius of the curve is 90 m and the coefficient of static friction is 0.50? Is it necessary to know the mass of the car to solve this problem?[10]

$$F_c = F_f$$
$$ma_c = \mu F_n$$
$$\frac{mv^2}{r} = \mu mg \qquad v = \sqrt{\mu g r} \qquad v = 21\ \text{m/s}$$

It is not necessary to know the mass.

11. A 1000-kg Indy car travels around a curve banked at 25° to the horizontal. If the radius of the curve is 80 m, at what speed must the car be travelling if no friction is present?[11]

Vertically:
$$F_n \cos\theta = ma_c$$
$$F_n = \frac{mg}{\cos\theta}$$

Horizontally:
$$F_c = F_n \sin\theta$$
$$ma_c = F_n \sin\theta$$

$$\left(\frac{mg}{\cos\theta}\right)\sin\theta = ma_c$$
$$g\tan\theta = \frac{v^2}{r}$$
$$v = \sqrt{rg\tan 25°}$$
$$v = 19\ \text{m/s}$$

[9] Physics Book Two, Irwin Publishing, Chapter 2 Conceptual Questions, #12
[10] Physics Book Two, Irwin Publishing, Chapter 2 Problems, #56
[11] Physics Book Two, Irwin Publishing, Chapter 2 Problems, #57

12. A 2.0-kg mass is attached to the end of a 3.0 m-long rope and spun in a vertical circle at a speed of 6.6 m/s. Determine the maximum and minimum tensions in the rope. [12]

Maximum tension occurs when the mass is at its lowest position. Tension acts upward, and gravity acts downward. The difference between these forces is the centripetal force:

$$T_{max} - mg = \frac{mv^2}{r}$$

$$T_{max} = \frac{mv^2}{r} + mg$$

$$T_{max} = \frac{(2.0\text{ kg})(6.6\text{ m/s})^2}{3.0\text{ m}} + (2.0\text{ kg})(9.8\text{ m/s}^2)$$

$$T_{max} = 49\text{ N}$$

The tension is minimized when the mass is at the top of its arc. Tension and gravity both act downward, and their sum is the centripetal force:

$$T_{min} + mg = \frac{mv^2}{r}$$

$$T_{min} = \frac{mv^2}{r} - mg$$

$$T_{min} = \frac{(2.0\text{ kg})(6.6\text{ m/s})^2}{3.0\text{ m}} - (2.0\text{ kg})(9.8\text{ m/s}^2)$$

$$T_{min} = 9.4\text{ N}$$

13. As a pilot comes out of a dive in a circular arc, she experiences an upward acceleration of 9.0 g's.
    a) If the pilot's mass is 60 kg, what is the magnitude of the force applied to her by her seat at the bottom of the arc?
    b) If the speed of the plane is 330 km/h, what is the radius of the plane's arc?[13]

a)

$$F_{net} = ma$$

$$F_n - mg = m(9g)$$

$$F_n = 9mg + mg$$

$$F_n = 10mg$$

$$F_n = 5.9 \times 10^3\text{ N}$$

b)

$$a_c = \frac{v^2}{r}$$

$$9g = \frac{v^2}{r}$$

$$r = \frac{v^2}{9g}$$

$$r = \frac{(91.67\text{ m/s})^2}{9(9.8\text{ m/s}^2)}$$

$$r = 95\text{ m}$$

14. A jet ($m = 2.00 \times 10^5$ kg), flying at 123 m/s, banks to make a horizontal circular turn. The radius of the turn is 3810 m. Calculate the necessary lifting force ***L***. [14]

REASONING Refer to Figure 5.10 in the text. The horizontal component of the lift **L** is the centripetal force that holds the plane in the circle. Thus,

$$L\sin\theta = \frac{mv^2}{r} \qquad (1)$$

The vertical component of the lift supports the weight of the plane; therefore,

$$L\cos\theta = mg \qquad (2)$$

[12] Physics Book Two, Irwin Publishing, Chapter 2 Problems, #60
[13] Physics Book Two, Irwin Publishing, Chapter 2 Problems, #61
[14] Physics, 7th Edition, Cutnell & Johnson, Chapter 5 Problems, #25

Dividing the first equation by the second gives

$$\tan\theta = \frac{v^2}{rg} \qquad (3)$$

Equation (3) can be used to determine the angle θ of banking. Once θ is known, then the magnitude of L can be found from either equation (1) or equation (2).

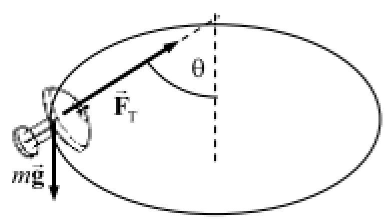
$$\theta = \tan^{-1}\left[\frac{(123\ \mathrm{m/s})^2}{(3810\ \mathrm{m})(9.80\ \mathrm{m/s^2})}\right] = 22.1°$$

The lifting force is, from equation (2),

$$L = \frac{mg}{\cos\theta} = \frac{(2.00\times10^5\ \mathrm{kg})(9.80\ \mathrm{m/s^2})}{\cos 22.1°} = \boxed{2.12\times10^6\ \mathrm{N}}$$

15. A train traveling at a constant speed rounds a curve of radius 235 m. A lamp suspended from the ceiling swings out to an angle of 17.5° throughout the curve. What is the speed of the train? [15]

The lamp must have the same speed and acceleration as the train. The forces on the lamp as the train rounds the corner are shown in the free-body diagram included. The tension in the suspending cord must not only hold the lamp up, but also provide the centripetal force needed to make the lamp move in a circle. Write Newton's 2nd law for the vertical direction, noting that the lamp is not accelerating vertically.



$$\sum F_y = F_T\cos\theta - mg = 0 \ \rightarrow\ F_T = \frac{mg}{\cos\theta}$$

The force moving the lamp in a circle is the horizontal portion of the tension. Write Newton's 2nd law for that radial motion.

$$\sum F_R = F_T\sin\theta = ma_R = mv^2/r$$

Substitute the expression for the tension from the first equation into the second equation, and solve for the speed.

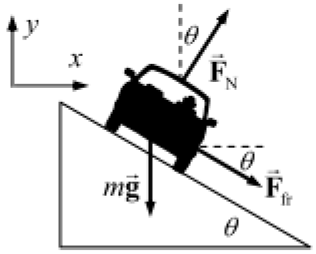
$$F_T\sin\theta = \frac{mg}{\cos\theta}\sin\theta = mg\tan\theta = mv^2/r \ \rightarrow$$

$$v = \sqrt{rg\tan\theta} = \sqrt{(235\ \mathrm{m})(9.80\ \mathrm{m/s^2})\tan 17.5°} = \boxed{26.9\ \mathrm{m/s}}$$

16. If a curve with a radius of 88 m is perfectly banked for a car traveling at 75 km/h, what must be the coefficient of static friction for a car not to skid when traveling at 95 km/h? [16]

[15] Physics 6th Edition, Giancoli, Chapter 5 Problems, #79
[16] Physics 6th Edition, Giancoli, Chapter 5 Problems, #21

Since the curve is designed for 75 km/h, traveling at a higher speed with the same radius means that more centripetal force will be required. That extra centripetal force will be supplied by a force of static friction, downward along the incline. See the free-body diagram for the car on the incline. Note that from Example 5-7 in the textbook, the no-friction banking angle is given by



$$\theta = \tan^{-1}\frac{v^2}{rg} = \tan^{-1}\frac{\left[(75\,\text{km/h})\left(\frac{1.0\,\text{m/s}}{3.6\,\text{km/h}}\right)\right]^2}{(88\text{ m})(9.8\,\text{m/s}^2)} = 26.7^\circ$$

Write Newton's 2nd law in both the x and y directions. The car will have no acceleration in the y direction, and centripetal acceleration in the x direction. We also assume that the car is on the verge of skidding, so that the static frictional force has its maximum value of $F_{fr} = \mu_s F_N$. Solve each equation for the normal force.

$$\sum F_y = F_N\cos\theta - mg - F_{fr}\sin\theta = 0 \;\rightarrow\; F_N\cos\theta - \mu_s F_N\sin\theta = mg \;\rightarrow$$

$$F_N = \frac{mg}{(\cos\theta - \mu_s\sin\theta)}$$

$$\sum F_x = F_N\sin\theta + F_{fr}\cos\theta = F_R = mv^2/r \;\rightarrow\; F_N\sin\theta + \mu_s F_N\cos\theta = mv^2/r \;\rightarrow$$

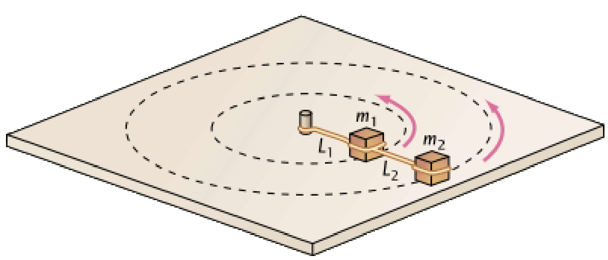
$$F_N = \frac{mv^2/r}{(\sin\theta + \mu_s\cos\theta)}$$

Equate the two expressions for F_N, and solve for the coefficient of friction. The speed of rounding the curve is given by $v = (95\,\text{km/h})\left(\frac{1.0\,\text{m/s}}{3.6\,\text{km/h}}\right) = 26.39\,\text{m/s}$.

$$\frac{mg}{(\cos\theta - \mu_s\sin\theta)} = \frac{mv^2/r}{(\sin\theta + \mu_s\cos\theta)} \;\rightarrow$$

$$\mu_s = \frac{\left(\frac{v^2}{r}\cos\theta - g\sin\theta\right)}{\left(g\cos\theta + \frac{v^2}{r}\sin\theta\right)} = \frac{\left(\frac{v^2}{r} - g\tan\theta\right)}{\left(g + \frac{v^2}{r}\tan\theta\right)} = \frac{\left(\frac{(26.39\,\text{m/s})^2}{88\text{ m}} - (9.8\,\text{m/s}^2)\tan 26.7^\circ\right)}{\left(9.8\,\text{m/s}^2 + \frac{(26.39\,\text{m/s})^2}{88\text{ m}}\tan 26.7^\circ\right)} = \boxed{0.22}$$

17. A block of mass m_1 is attached to a rope of length L_1, which is fixed at one end to a table. The mass moves in a horizontal circle supported by a frictionless table. A second block of mass m_2 is attached to the first mass by a rope of length L_2. This mass also moves in a circle, as shown below. If the period of the motion is T, find the tension in each rope (assume all ropes are massless).[17]



[17] Physics Book Two, Irwin Publishing, Chapter 2 Problems, #63

On mass 2:

$$F_c = m_2\left(\frac{4\pi^2 r}{T^2}\right)$$

$$T_2 = m_2\left(\frac{4\pi^2 r}{T^2}\right)$$

$$T_2 = m_2\left(\frac{4\pi^2 (L_1 + L_2)}{T^2}\right)$$

On mass 1:

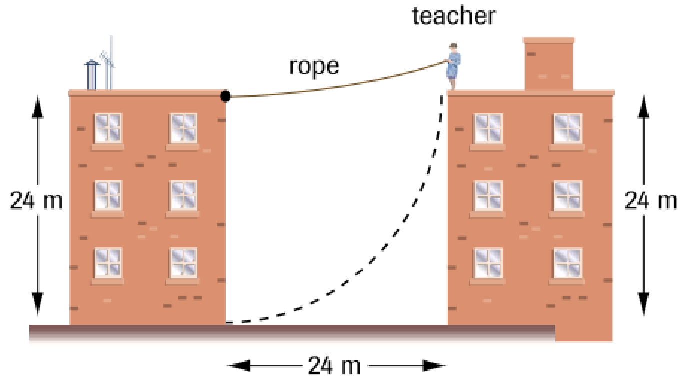
$$F_c = m_1\left(\frac{4\pi^2 r}{T^2}\right)$$

$$T_1 - T_2 = m_1\left(\frac{4\pi^2 r}{T^2}\right)$$

$$T_1 = m_1\left(\frac{4\pi^2 L_1}{T^2}\right) + m_2\left(\frac{4\pi^2 (L_1 + L_2)}{T^2}\right)$$

$$T_1 = \left(\frac{4\pi^2}{T^2}\right)(m_1 L_1 + m_2(L_1 + L_2))$$

18. Your favourite physics teacher who is late for class attempts to swing from the roof of a 24-m high building to the bottom of an identical building using a 24-m rope as shown in Figure 5. She starts from rest with the rope horizontal, but the rope will break if the tension force in it is twice the weight of the teacher. How high is the swinging physicist above level when the rope breaks? (Hint: Apply the law of conservation of energy.)[18]

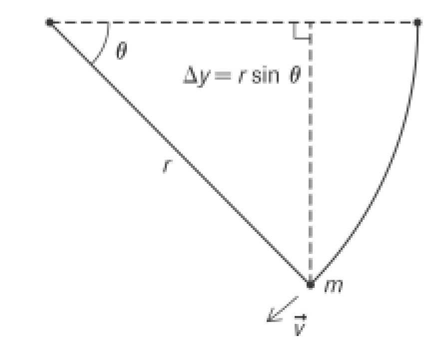


$r = 24$ m
$F_T = 2mg$
$v_i = 0$

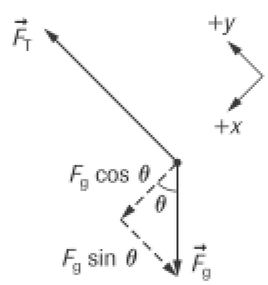
Note: If you assign this question at this stage, give your students the hint that they must apply the law of conservation of energy, which they studied in the previous grade. As the teacher in the question drops a vertical distance Δy, she gains an amount of kinetic energy equal to the amount of gravitational potential energy she loses.

The first figure shows the system diagram after the teacher has swung downward and the rope is at an angle θ to the horizontal. The second figure shows the corresponding FBD of the teacher at that instant.

(a) The system diagram



(b) The FBD



[18] Physics 12, Nelson Education, Chapter 3 Review, #27

Considering the y-components of the forces:

$$\Sigma F_y = ma_y$$
$$F_T - mg\sin\theta = \frac{mv^2}{r}$$
$$2mg - mg\sin\theta = \frac{mv^2}{r}$$
$$2g - g\sin\theta = \frac{v^2}{r} \quad \text{(Equation 1)}$$

Applying the law of conservation of energy:

$$E_K = \Delta E_P$$
$$\frac{mv^2}{2} = mgr\sin\theta$$
$$\frac{v^2}{r} = 2g\sin\theta \quad \text{(Equation 2)}$$

Substituting Equation 2 into Equation 1:

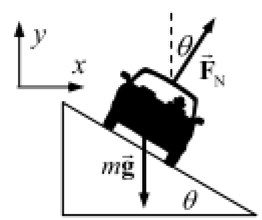
$$2g - g\sin\theta = 2g\sin\theta$$
$$2 - \sin\theta = 2\sin\theta$$
$$2 = 3\sin\theta$$
$$\sin\theta = \frac{2}{3}$$

Finally,

$$\Delta y = r\sin\theta$$
$$= (24\text{ m})\left(\frac{2}{3}\right)$$
$$\Delta y = 16\text{ m}$$

Thus, the teacher has dropped by 16 m, and so is 8.0 m above the ground.

19. A 1200-kg car rounds a curve of radius 67 m banked at an angle of 12°. If the car is traveling at 95 km/h, will friction be required? If so, how much and in what direction?[19]



The car moves in a horizontal circle, and so there must be a net horizontal centripetal force. The car is not accelerating vertically. Write Newton's 2nd law for both the x and y directions.

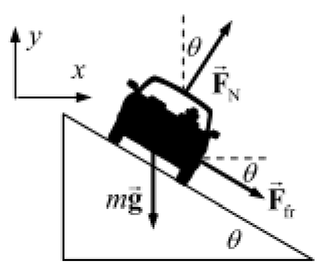
$$\sum F_y = F_N\cos\theta - mg = 0 \rightarrow F_N = \frac{mg}{\cos\theta}$$
$$\sum F_x = \sum F_R = F_N\sin\theta = ma_x$$

The amount of centripetal force needed for the car to round the curve is

$$F_R = mv^2/r = (1200\text{ kg})\frac{\left[(95\text{ km/h})\left(\frac{1.0\text{ m/s}}{3.6\text{ km/h}}\right)\right]^2}{67\text{ m}} = 1.247\times10^4\text{ N}.$$

The actual horizontal force available from the normal force is

$$F_N\sin\theta = \frac{mg}{\cos\theta}\sin\theta = mg\tan\theta = (1200\text{ kg})(9.80\text{ m/s}^2)\tan 12^\circ = 2.500\times10^3\text{ N}.$$



Thus more force is necessary for the car to round the curve than can be supplied by the normal force. That extra force will have to have a horizontal component to the right in order to provide the extra centripetal force. Accordingly, we add a frictional force pointed down the plane. That corresponds to the car not being able to make the curve without friction.

Again write Newton's 2nd law for both directions, and again the y acceleration is zero.

$$\sum F_y = F_N\cos\theta - mg - F_{fr}\sin\theta = 0 \rightarrow F_N = \frac{mg + F_{fr}\sin\theta}{\cos\theta}$$
$$\sum F_x = F_N\sin\theta + F_{fr}\cos\theta = mv^2/r$$

[19] Physics 6th Edition, Giancoli, Chapter 5 Problems, #22

Substitute the expression for the normal force from the y equation into the x equation, and solve for the friction force.

$$\frac{mg + F_{fr}\sin\theta}{\cos\theta}\sin\theta + F_{fr}\cos\theta = mv^2/r \quad \rightarrow \quad (mg + F_{fr}\sin\theta)\sin\theta + F_{fr}\cos^2\theta = m\frac{v^2}{r}\cos\theta$$

$$F_{fr} = m\frac{v^2}{r}\cos\theta - mg\sin\theta = (1.247\times10^4\ \text{N})\cos12^\circ - (1200\ \text{kg})(9.80\ \text{m/s}^2)\sin12^\circ$$

$$= 9.752\times10^3\ \text{N}$$

So a frictional force of $\boxed{9.8\times10^3\ \text{N down the plane}}$ is needed to provide the necessary centripetal force to round the curve at the specified speed.