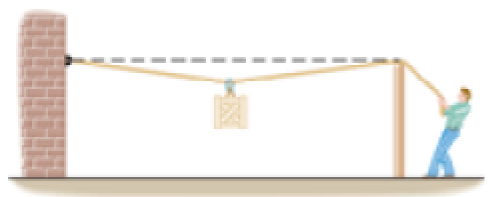
Conceptual Questions

1. A 100-kg sofa needs to be moved across a level floor. The coefficient of static friction between the sofa and the floor is 0.40. Two physics students decide to apply a force ***F*** on the sofa. One student recommends that the force be applied upward at an angle ϑ above the horizontal. The other student recommends that the force be applied downward at an angle ϑ below the horizontal. Explain which student has the better ideas and why.[1]

The student who wants to apply the force above the horizontal has the better idea. The horizontal component of the applied force in the direction of motion will be the same regardless of whether the force is applied above or below the horizontal. It is in the students' best interest to minimize the amount of friction. Recall that the frictional force is directly proportional to the normal force. If they apply the force above the horizontal, this will reduce the magnitude of the normal force needed to be supplied by the floor on the sofa, which will therefore reduce the frictional force and make it easier to move the sofa. On the other hand, if they apply the force below the horizontal, this will increase the normal force required and thereby increase the frictional force, making it harder to move the sofa.

2. A weight hangs from a ring at the middle of a rope, as the drawing illustrates. Can the person who is pulling on the right end of the rope ever make the rope perfectly horizontal? Explain your answer in terms of the forces that act on the ring.[2]



No. In order to prevent the box from accelerating downwards, the rope must cancel out the force of gravity. The only way it can do this is to have some component of its tension force in the vertical direction, which means the rope must pull at angle above the horizontal, and so cannot be perfectly horizontal.

3. An object is held in place by friction on an inclined surface. The angle of inclination is increased very slowly until the object just starts moving. If the surface is kept at this angle, the object[3]

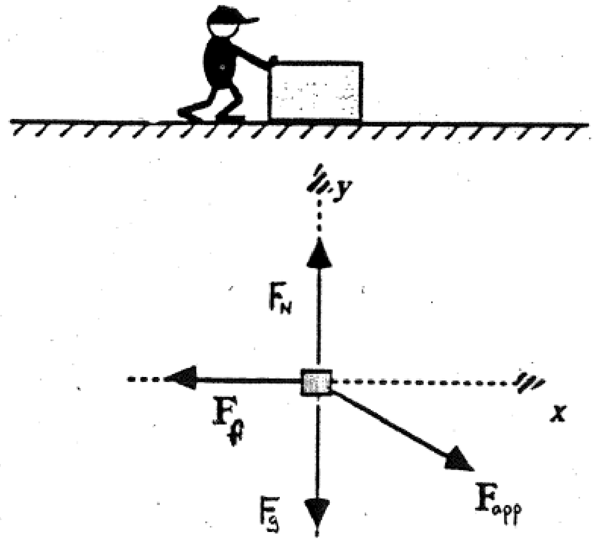
a) slows down	c) moves at uniform speed
b) speeds up	d) none of the above

Ans: B

The force of gravity parallel to the ramp has increased sufficiently to overcome the force of static friction acting on the object. Since the object is now moving, the static force of friction has changed to a kinetic force of friction, which is smaller in magnitude. This means that there exists a non-zero force acting down the ramp, so the object will speed up.

[1] Physics Book Two, Irwin Publishing, Chapter 2 Conceptual Questions, #7
[2] Physics, 7th Edition, Cutnell & Johnson, Chapter 4 Conceptual Questions, #26
[3] Peer Instruction – A User's Guide, Mazur, Forces CT 9

4. A person pushes a crate so that it moves at constant velocity toward the right. A free body diagram for the crate is shown (forces not to scale). Which choice below represents the relative magnitudes of the forces?[4]
   a) $F_{app} = F_f$ and $F_N = F_g$
   b) $F_{app} = F_f$ and $F_N > F_g$
   c) $F_{app} = F_f$ and $F_N < F_g$
   d) $F_{app} > F_f$ and $F_N = F_g$
   e) $F_{app} > F_f$ and $F_N > F_g$



ANS: E

The horizontal component of the applied force equals the force of friction since the box moves at a constant speed. Since the applied force also has a vertical component, the total magnitude must be larger than the magnitude of friction.

In the vertical direction there is no acceleration, so the normal force must cancel both the force of gravity and the vertical component of the applied force. Therefore the magnitude of the normal force is larger than that of the force of gravity.

5. An object sliding up an inclined plane will slow down as a result of both the force of friction and the force of gravity. As the angle of inclination of the ramp increases, the component of the force of gravity along the ramp increases, but the friction force will decrease (since the normal force decreases). If the coefficient of kinetic friction between the object and the ramp is 1.000, which of the following situations would bring the object to rest in the least amount of time?[5]
   a) Small angles of ramp inclination
   b) Large angles of ramp inclination
   c) 45° ramp inclination
   d) Deceleration is constant for all angles of ramp inclination

ANS: C

The acceleration of the block in this case (μ = 1.00) depends on $g(\cos\vartheta + \sin\vartheta)$. When $\vartheta = 45°$, the result is maximized (When the angle ϑ increases, the $\sin\vartheta$ term increases as the $\cos\vartheta$ decreases (and vice-versa), but their sum does not remain constant since these functions do not vary linearly with ϑ.).

Problems

6. Three movers are applying forces $\mathbf{F_1}$ = 100 N [W20°N], $\mathbf{F_2}$ = 200 N [E40°S], and $\mathbf{F_3}$ = 300 N [S] on a 300-kg grand piano. If μ_K for the piano is 0.10, determine
   a) the net force acting on the piano
   b) the acceleration of the piano[6]

[4] Peer Instruction – A User's Guide, Mazur
[5] Almeida, F., Physics Department, Victoria Park C.I.
[6] Physics Book Two, Irwin Publishing, Chapter 2 Problems, #38

a) $\vec{F}_{net} = \vec{F}_a - \vec{F}_f$

The sum of the x components is:

$\vec{F}_{a_x} = \vec{F}_{1_x} + \vec{F}_{2_x} + \vec{F}_{3_x}$

$\vec{F}_{a_x} = (100\text{ N})\cos 20°\ [W] + (200\text{ N})\cos 40°\ [E]$

$\vec{F}_{a_x} = 59\text{ N [E]}$

The sum of the y components is:

$\vec{F}_{a_y} = \vec{F}_{1_y} + \vec{F}_{2_y} + \vec{F}_{3_y}$

$\vec{F}_{a_y} = (100\text{ N})\sin 20°\ [N] + (200\text{ N})\sin 40°\ [S] + 300\text{ N [S]}$

$\vec{F}_{a_y} = 394\text{ N [S]}$

$F_a = \sqrt{F_{a_x}{}^2 + F_{a_y}{}^2}$

$F_a = \sqrt{(59\text{ N})^2 + (394\text{ N})^2}$

$F_a = 399\text{ N}$

$\tan\theta = \dfrac{F_{a_x}}{F_{a_y}}$

$\theta = \tan^{-1}\left(\dfrac{394\text{ N}}{59\text{ N}}\right)$

$\theta = 8.5°$

$\vec{F}_a = 399\text{ N [S8.5°E]}$

$F_f = \mu_k F_n$

$F_f = \mu_k mg$

$F_f = (0.10)(300\text{ kg})(9.8\text{ m/s}^2)$

$F_f = 294\text{ N}$

$\vec{F}_f = 294\text{ N [N8.5°W]}$

$\vec{F}_{net} = \vec{F}_a + \vec{F}_f$

$\vec{F}_{net} = 399\text{ N [S8.5°E]} + 294\text{ N [N8.5°W]}$

$\vec{F}_{net} = 399\text{ N [S8.5°E]} - 294\text{ N [S8.5°E]}$

$\vec{F}_{net} = 105\text{ N [S8.5°E]}$

The net force is $\vec{F}_{net} = 105\text{ N [S8.5°E]}$

b) $\vec{F}_{net} = m\vec{a}$

$\vec{a} = \dfrac{\vec{F}_{net}}{m}$

$\vec{a} = \dfrac{105\text{ N [S8.5°E]}}{300\text{ kg}}$

$\vec{a} = 0.35\text{ m/s}^2\text{ [S8.5°E]}$

7. A worker drags a 20-kg bag of cement across a floor by applying a force of 100 N at an angle of 50° to the horizontal. If the coefficient of kinetic friction between the cement bag and the floor is 0.30, determine the acceleration of the bag.[7]

$\vec{F}_{net} = \vec{F}_{\text{applied force in the x direction}} + \vec{F}_{\text{kinetic friction}}$

$\vec{F}_{net} = \vec{F}_{a_x} + \vec{F}_k$

Find the kinetic frictional force, F_k:

$F_k = \mu_k F_n$

$F_k = (0.30)(F_g - F_a \sin 50°)$

$F_k = (0.30)[(20\text{ kg})(9.8\text{ m/s}^2) - (100\text{ N})\sin 50°]$

$F_k = 36\text{ N}$

$F_{net} = (100\text{ N})\cos 50° - 36\text{ N}$

$F_{net} = 28\text{ N}$

$F_{net} = ma$

$28\text{ N} = ma$

$a = \dfrac{28\text{ N}}{20\text{ kg}}$

$a = 1.4\text{ m/s}^2$

8. While mopping the deck, a sailor pushes with a force of 30 N down on the handle of his mop at an angle of 45° to the horizontal. If the mop accelerates horizontally at 1.0 m/^{s} and the coefficient of kinetic friction is 0.10, what is the mass of the mop?[8]

$F_{net} = F_a - F_k$

The horizontal acceleration of 1.0 m/s^2 is the net acceleration of the mop, therefore:

$F_{net} = ma_x$

$ma_x = F_{a_x} - \mu_k F_n$

$ma_x = (30\text{ N})\cos 45° - [(0.1)(F_g + F_a \sin 45°)]$

$ma_x = 21.2\text{ N} - [(0.1)(mg + 21.2\text{ N})]$

$ma_x = 19.09\text{ N} - 0.1mg$

$(1.0\text{ m/s}^2)m = 19.09\text{ N} - 0.1mg$

$m(1.0\text{ m/s}^2 + 0.1g) = 19.09\text{ N}$

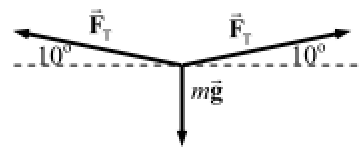
$m(1.98\text{ m/s}^2) = 19.09\text{ N}$

$m = 9.6\text{ kg}$

[7] Physics Book Two, Irwin Publishing, Chapter 2 Problems, #39

[8] Physics Book Two, Irwin Publishing, Chapter 2 Problems, #43

9. Arlene is to walk across a "high wire" strung horizontally between two buildings 10.0 m apart. The sag in the rope when she is at the midpoint is 10.0°, as shown. If her mass is 50.0 kg, what is the tension in the rope at this point?[9]

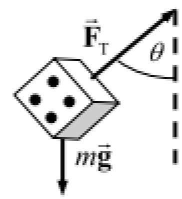


Consider the point in the rope directly below Arlene. That point can be analyzed as having three forces on it – Arlene's weight, the tension in the rope towards the right point of connection, and the tension in the rope towards the left point of connection. Assuming the rope is massless, those two tensions will be of the same magnitude. Since the point is not accelerating the sum of the forces must be zero. In particular, consider the sum of the vertical forces on that point, with UP as the positive direction.

$$\sum F = F_T \sin 10.0^\circ + F_T \sin 10.0^\circ - mg = 0 \quad \rightarrow$$

$$F_T = \frac{mg}{2\sin 10.0^\circ} = \frac{(50.0\text{ kg})(9.80\text{ m/s}^2)}{2\sin 10.0^\circ} = \boxed{1.41\times10^3\text{ N}}$$

10. A pair of fuzzy dice is hanging by a string from your rearview mirror. While you are accelerating from a stoplight to 28 m/s in 6.0 s, what angle ϑ does the string make with the vertical?[10]



Consider a free-body diagram of the dice. The car is moving to the right. The acceleration of the dice is found from Eq. 2-11a.

$$v = v_0 + = a_x t \quad \rightarrow \quad a_x = \frac{v - v_0}{t} = \frac{28\text{ m/s} - 0}{6.0\text{ s}} = 4.67\text{ m/s}^2$$

Now write Newton's 2nd law for both the vertical (y) and horizontal (x) directions.

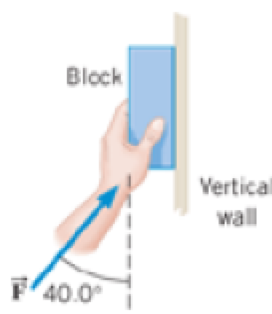
$$\sum F_y = F_T\cos\theta - mg = 0 \quad \rightarrow \quad F_T = \frac{mg}{\cos\theta} \qquad \sum F_x = F_T \sin\theta = ma_x$$

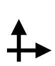
Substitute the expression for the tension from the y equation into the x equation.

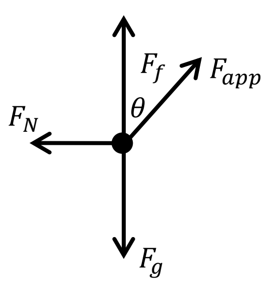
$$ma_x = F_T \sin\theta = \frac{mg}{\cos\theta}\sin\theta = mg\tan\theta \quad \rightarrow \quad a_x = g\tan\theta$$

$$\theta = \tan^{-1}\frac{a_x}{g} = \tan^{-1}\frac{4.67\text{ m/s}^2}{9.8\text{ m/s}^2} = \boxed{25^\circ}$$

11. The weight of the block in the drawing is 88.9 N. The coefficient of static friction between the block and the vertical wall is 0.560.
   a) What minimum force is required to prevent the block from sliding down the wall?
   b) What minimum force is required to start the block moving up the wall?[11]



a) 



$$\sum F_x = F_{app}\sin\theta - F_N = 0$$
$$\sum F_y = F_{app}\cos\theta + F_f - F_g = 0$$

[9] Physics 6th Edition, Giancoli, Chapter 4 Problems, #23
[10] Physics 6th Edition, Giancoli, Chapter 4 Problems, #32
[11] Physics, 7th Edition, Cutnell & Johnson, Chapter 4 Problems, #60

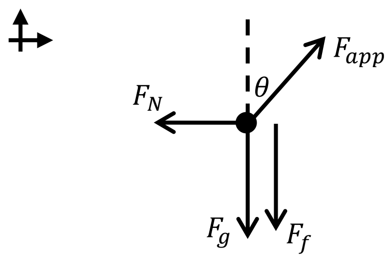
From x-direction:

$$F_N = F_{app}\sin\theta$$

From y-direction:

$$F_{app}\cos\theta + \mu F_N - F_g = 0$$
$$F_{app}\cos\theta + \mu(F_{app}\sin\theta) - F_g = 0$$
$$F_{app} = \frac{F_g}{\cos\theta + \mu\sin\theta}$$
$$F_{app} = \frac{88.9\ N}{\cos 40° + 0.560(\sin 40°)}$$
$$F_{app} = 78.952\ N$$

A minimum force of 79.0 N [fwd40°up] is required to prevent the block from sliding.

b) 

$$\sum F_x = F_{app}\sin\theta - F_N = 0$$
$$\sum F_y = F_{app}\cos\theta - F_f - F_g = 0$$

From x-direction:

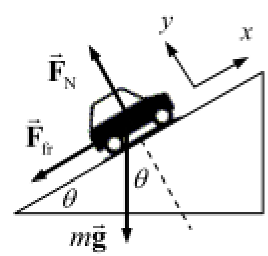
$$F_N = F_{app}\sin\theta$$

From y-direction:

$$F_{app}\cos\theta - \mu F_N - F_g = 0$$
$$F_{app}\cos\theta - \mu(F_{app}\sin\theta) - F_g = 0$$
$$F_{app} = \frac{F_g}{\cos\theta - \mu\sin\theta}$$
$$F_{app} = \frac{88.9\ N}{\cos 40° - 0.560(\sin 40°)}$$
$$F_{app} = 218.92\ N$$

A minimum force of 219 N [fwd40°up] is required to start the block moving.

12. The coefficient of static friction between hard rubber and normal street pavement is about 0.8. On how steep a hill (maximum angle) can you leave a car parked?[12]



See the included free-body diagram. To find the maximum angle, assume that the car is just ready to slide, so that the force of static friction is a maximum. Write Newton's 2nd law for both directions. Note that for both directions, the net force must be zero since the car is not accelerating.

$$\sum F_y = F_N - mg\cos\theta = 0 \rightarrow F_N = mg\cos\theta$$
$$\sum F_x = mg\sin\theta - F_{fr} = 0 \rightarrow mg\sin\theta = F_{fr} = \mu_s F_N = \mu_s mg\cos\theta$$
$$\mu_s = \frac{mg\sin\theta}{mg\cos\theta} = \tan\theta = 0.8 \rightarrow \theta = \tan^{-1}0.8 = 39° = \boxed{40°} \quad (1 \text{ sig fig})$$

13. A rescue worker slides a box of supplies from rest down a hill to a group of trapped campers. The hill is inclined at 25° to the horizontal and is 200 m long. If the coefficient of kinetic friction on the hill is 0.45,
a) what is the acceleration of the box as it goes down the hill?
b) at what speed does the box reach the bottom of the hill?[13]

[12] Physics 6th Edition, Giancoli, Chapter 4 Problems, #40
[13] Physics Book Two, Irwin Publishing, Chapter 2 Problems, #46

a) $F_{net} = F_x - F_k$

$ma = mg\sin\theta - \mu_k F_n$

$ma = mg\sin\theta - \mu_k(mg\cos\theta)$

$a = g\sin\theta - \mu_k g\cos\theta$

$a = (9.8\ \text{m/s}^2)\sin 25° - (0.45)(9.8\ \text{m/s}^2)\cos 25°$

$a = 0.14\ \text{m/s}^2$

The acceleration of the box is 0.14 m/s².

b) $v_f^2 = v_i^2 + 2ad$

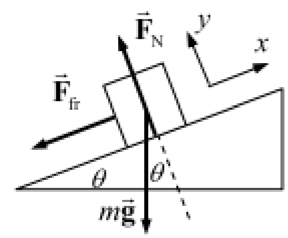
$v_f^2 = 2(0.14\ \text{m/s}^2)(200\ \text{m})$

$v_f = 7.6\ \text{m/s}$

The box reaches the bottom of the hill at 7.6 m/s².

14. A small block of mass ***m*** is given an initial speed ***v₀*** up a ramp inclined at an angle ***ϑ*** to the horizontal. It travels a distance ***d*** up the ramp and comes to rest. Determine a formula for the coefficient of kinetic friction between the block and ramp.[14]

We derive two expressions for acceleration – one from the kinematics, and one from the dynamics. From Eq. 2-11c with a starting speed of v_o up the plane and a final speed of zero, we have



$$v^2 - v_o^2 = 2a(x - x_0) \quad \rightarrow \quad a = \frac{-v_o^2}{2(x-x_0)} = \frac{-v_o^2}{2d}$$

Write Newton's 2nd law for both the *x* and *y* directions. Note that the net force in the *y* direction is zero, since the block does not accelerate in the *y* direction.

$$\sum F_y = F_N - mg\cos\theta = 0 \quad \rightarrow \quad F_N = mg\cos\theta$$

$$\sum F_x = -mg\sin\theta - F_{fr} = ma \quad \rightarrow \quad a = \frac{-mg\sin\theta - F_{fr}}{m}$$

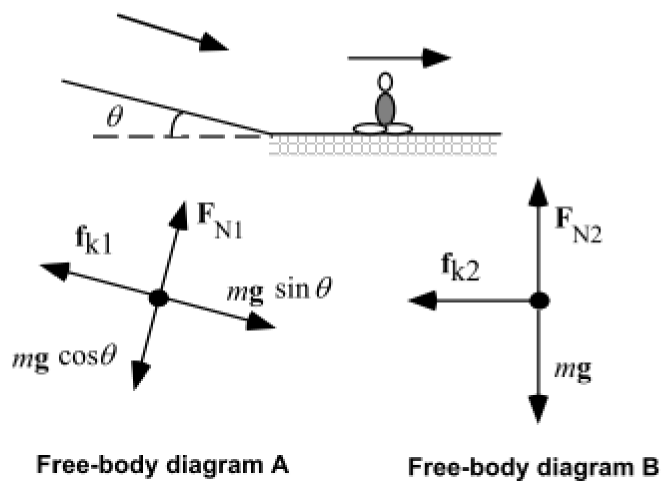
Now equate the two expressions for the acceleration, substitute in the relationship between the frictional force and the normal force, and solve for the coefficient of friction.

$$a = \frac{-mg\sin\theta - F_{fr}}{m} = \frac{-v_o^2}{2d} \quad \rightarrow \quad \frac{mg\sin\theta + \mu_k mg\cos\theta}{m} = \frac{v_o^2}{2d} \quad \rightarrow$$

$$\boxed{\mu_k = \frac{v_o^2}{2gd\cos\theta} - \tan\theta}$$

15. A penguin slides at a constant velocity if 1.4 m/s down an icy incline. The incline slopes above the horizontal at an angle of 6.9°. At the bottom of the incline, the penguin slides onto a horizontal patch of ice. The coefficient of kinetic friction between the penguin and the ice is the same for the incline as for the horizontal patch. How much time is required for the penguin to slide to a halt after entering the horizontal patch of ice?[15]

[14] Physics 6th Edition, Giancoli, Chapter 4 Problems, #60
[15] Physics, 7th Edition, Cutnell & Johnson, Chapter 4 Problems, #85



REASONING AND SOLUTION The penguin comes to a halt on the horizontal surface because the kinetic frictional force opposes the motion and causes it to slow down. The time required for the penguin to slide to a halt (v = 0 m/s) after entering the horizontal patch of ice is, according to Equation 2.4,

$$t = \frac{v - v_0}{a_x} = \frac{-v_0}{a_x}$$

We must, therefore, determine the acceleration of the penguin as it slides along the horizontal patch.

For the penguin sliding on the horizontal patch of ice, we find from free-body diagram B and Newton's second law in the x direction (motion to the right is taken as positive) that

$$\sum F_x = -f_{\text{k2}} = ma_x \qquad \text{or} \qquad a_x = \frac{-f_{\text{k2}}}{m} = \frac{-\mu_{\text{k}} F_{\text{N2}}}{m}$$

In the y direction in free-body diagram B, we have $\sum F_y = F_{\text{N2}} - mg = 0$, or $F_{\text{N2}} = mg$. Therefore, the acceleration of the penguin is

$$a_x = \frac{-\mu_{\text{k}} mg}{m} = -\mu_{\text{k}} g \qquad (1)$$

Equation (1) indicates that, in order to find the acceleration a_x, we must find the coefficient of kinetic friction.

We are told in the problem statement that the coefficient of kinetic friction between the penguin and the ice is the same for the incline as for the horizontal patch. Therefore, we can use the motion of the penguin on the incline to determine the coefficient of friction and use it in Equation (1).

For the penguin sliding down the incline, we find from free-body diagram A and Newton's second law (taking the direction of motion as positive) that

$$\sum F_x = mg\sin\theta - f_{\text{k1}} = ma_x = 0 \qquad \text{or} \qquad f_{\text{k1}} = mg\sin\theta \qquad (2)$$

Here, we have used the fact that the penguin slides down the incline with a constant velocity, so that it has zero acceleration. From Equation 4.8, we know that $f_{k1} = \mu_k F_{N1}$. Applying Newton's second law in the direction perpendicular to the incline, we have

$$\Sigma F_y = F_{N1} - mg\cos\theta = 0 \qquad \text{or} \qquad F_{N1} = mg\cos\theta$$

Therefore, $f_{k1} = \mu_k mg\,\cos\theta$, so that according to Equation (2), we find

$$f_{k1} = \mu_k mg\cos\theta = mg\sin\theta$$

Solving for the coefficient of kinetic friction, we have

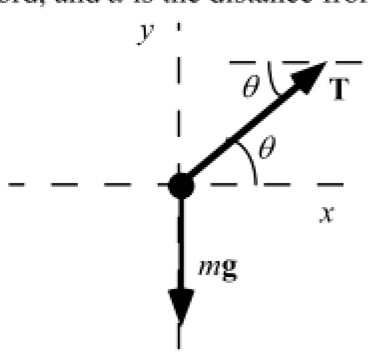
$$\mu_k = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

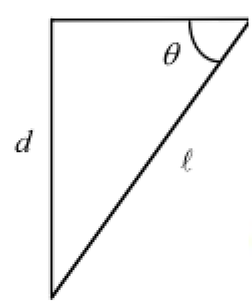
Finally, the time required for the penguin to slide to a halt after entering the horizontal patch of ice is

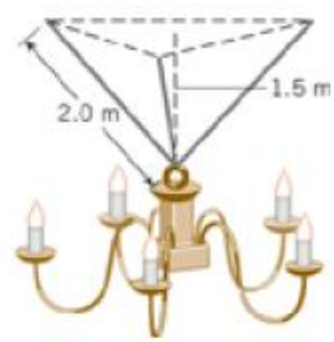
$$t = \frac{-v_0}{a_x} = \frac{-v_0}{-\mu_k g} = \frac{v_0}{g\tan\theta} = \frac{1.4\text{ m/s}}{(9.80\text{ m/s}^2)\tan\ 6.9°} = \boxed{1.2\text{ s}}$$

16. A 44-kg chandelier is suspended 1.5 m below a ceiling by three wires, each of which has the same tension and the same length of 2.0 m. Find the tension in each wire. [16]

REASONING There are four forces that act on the chandelier; they are the forces of tension T in each of the three wires, and the downward force of gravity mg. Under the influence of these forces, the chandelier is at rest and, therefore, in equilibrium. Consequently, the sum of the x components as well as the sum of the y components of the forces must each be zero. The figure below shows the free-body diagram for the chandelier and the force components for a suitable system of x, y axes. Note that the free-body diagram only shows one of the forces of tension; the second and third tension forces are not shown. The triangle at the right shows the geometry of one of the cords, where ℓ is the length of the cord, and d is the distance from the ceiling.







We can use the forces in the y direction to find the magnitude T of the tension in any one wire.

[16] Physics, 7th Edition, Cutnell & Johnson, Chapter 4 Problems, #57

SOLUTION Remembering that there are three tension forces, we see from the free-body diagram that

$$3T\sin\theta = mg \qquad \text{or} \qquad T = \frac{mg}{3\sin\theta} = \frac{mg}{3(d/\ell)} = \frac{mg\ell}{3d}$$

Therefore, the magnitude of the tension in any one of the cords is

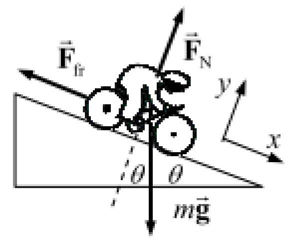
$$T = \frac{(44\text{ kg})(9.80\text{ m/s}^2)(2.0\text{ m})}{3(1.5\text{ m})} = \boxed{1.9\times10^2\text{ N}}$$

17. A bicyclist of mass 65 kg (including the bicycle) can coast down a 6.0° hill at a steady speed of 6.0 km/h because of air resistance. How much force must be applied to climb the hill at the same speed and same air resistance?[17]

Consider a free-body diagram for the cyclist coasting downhill at a constant speed. Since there is no acceleration, the net force in each direction must be zero. Write Newton's 2nd law for the x direction.

$$\sum F_x = mg\sin\theta - F_{fr} = 0 \;\rightarrow\; F_{fr} = mg\sin\theta$$

This establishes the size of the air friction force at 6.0 km/h, and so can be used in the next part.

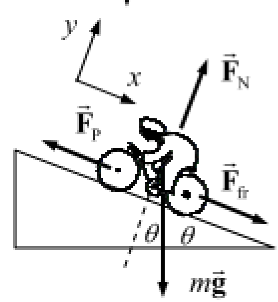


Now consider a free-body diagram for the cyclist climbing the hill. F_P is the force pushing the cyclist uphill. Again, write Newton's 2nd law for the x direction, with a net force of 0.

$$\sum F_x = F_{fr} + mg\sin\theta - F_P = 0 \;\rightarrow$$

$$F_P = F_{fr} + mg\sin\theta = 2mg\sin\theta$$

$$= 2(65\text{ kg})(9.8\text{ m/s}^2)(\sin 6.0^\circ) = \boxed{1.3\times10^2\text{ N}}$$



18. A While moving in, a new homeowner is pushing a box across the floor at a constant velocity. The coefficient of kinetic friction between the box and the floor is 0.41. The pushing force is directed downward at an angle ***ϑ*** below the horizontal. When ***ϑ*** is greater than a certain value, it is not possible to move the box, ***no matter how large the pushing force is***. Find that value of ***ϑ***.[18]

If the box won't move no matter how large the pushing force is, this means that the horizontal applied force must be negated by the extra friction force created when pushing downwards, such that pushing has no net effect. So,

$$F_{app}\cos\theta = \mu F_{app}\sin\theta$$

$$\tan\theta = \frac{1}{\mu}$$

$$\theta = \tan^{-1}\left(\frac{1}{0.41}\right)$$

$$\theta = 68^\circ$$

[17] Physics 6th Edition, Giancoli, Chapter 4 Problems, #65
[18] Physics, 7th Edition, Cutnell & Johnson, Chapter 4 Problems, #111