

**Conceptual Questions**

1. During baseball practice, a batter hits a very high fly ball and then runs in a straight line and catches it. Which had the greater displacement, the batter or the ball? Explain.<sup>1</sup>

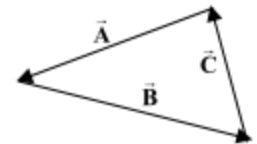
Since both the batter and the ball started their motion at the same location (where the ball was hit) and ended their motion at the same location (where the ball was caught), the displacement of both was the same.

2. Can the displacement vector for a particle moving in two dimensions ever be longer than the path-length travelled by the particle over the same time interval? Can it ever be less? Discuss.<sup>2</sup>

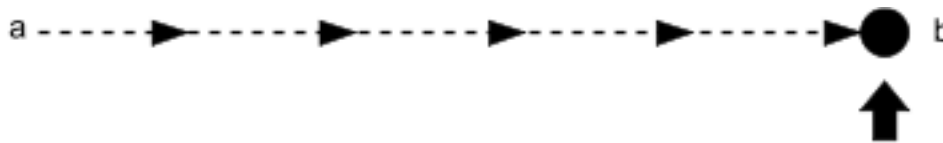
The displacement can be thought of as the “straight line” path from the initial location to the final location. The length of path will always be greater than or equal to the displacement, because the displacement is the shortest distance between the two locations. Thus the displacement can never be longer than the length of path, but it can be less. For any path that is not a single straight line segment, the length of path will be longer than the displacement.

3. Can two vectors of unequal magnitude add up to give the zero vector? Can *three* unequal vectors? Under what conditions?<sup>3</sup>

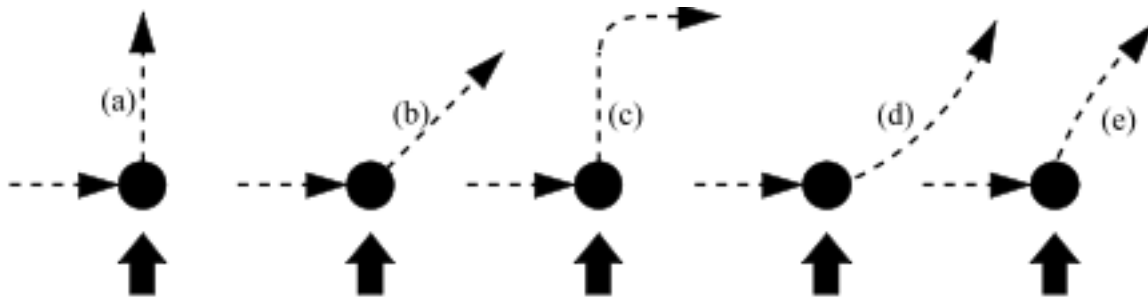
Two vectors of unequal magnitude can never add to give the zero vector. However, three vectors of unequal magnitude can add to give the zero vector. If their geometric sum using the tail-to-tip method gives a closed triangle, then the vector sum will be zero. See the diagram, in which  $\vec{A} + \vec{B} + \vec{C} = \vec{0}$ .



4. The figure depicts a hockey puck sliding with constant speed  $v_0$  in a straight line from point “a” to point “b” on a frictionless horizontal surface. Forces exerted by the air are negligible. You are looking down on the puck. When the puck reaches point “b”, it receives a swift horizontal kick in the direction of the heavy print arrow. Had the puck been at rest at point “b”, then the kick would have set the puck in horizontal motion with a speed  $v_k$  in the direction of the kick.<sup>4</sup>



- a) Which of the paths below would the puck most closely follow after receiving the kick?



<sup>1</sup> Physics 6<sup>th</sup> Edition, Giancoli, Chapter 3 Questions, #4

<sup>2</sup> Physics 6<sup>th</sup> Edition, Giancoli, Chapter 3 Questions, #3

<sup>3</sup> Physics 6<sup>th</sup> Edition, Giancoli, Chapter 3 Questions, #7

<sup>4</sup> Force Concept Inventory, Hestenes, Halloun, Wells, and Swackhamer, #8, 9

ANS: **B**

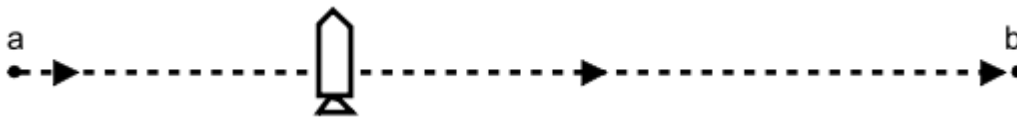
**C**, **D** and **E** describe curved paths which indicate accelerated motion over time. Since the kick was “swift” the force on the puck, and therefore its acceleration was instantaneous. The horizontal motion in **A** is cancelled out, which would imply a force to stop it in the horizontal direction, which did not occur.

- b) The speed of the puck just after it receives the kick is:
- (a) equal to the speed “ $v_0$ ” it had before it received the kick.
  - (b) equal to the speed “ $v_k$ ” resulting from the kick and independent of the speed “ $v_0$ ”.
  - (c) equal to the arithmetic sum of the speeds “ $v_0$ ” and “ $v_k$ ”.
  - (d) smaller than either the speeds “ $v_0$ ” or “ $v_k$ ”
  - (e) greater than either of the speeds “ $v_0$ ” or “ $v_k$ ”, but less than the arithmetic sum of these two speeds.

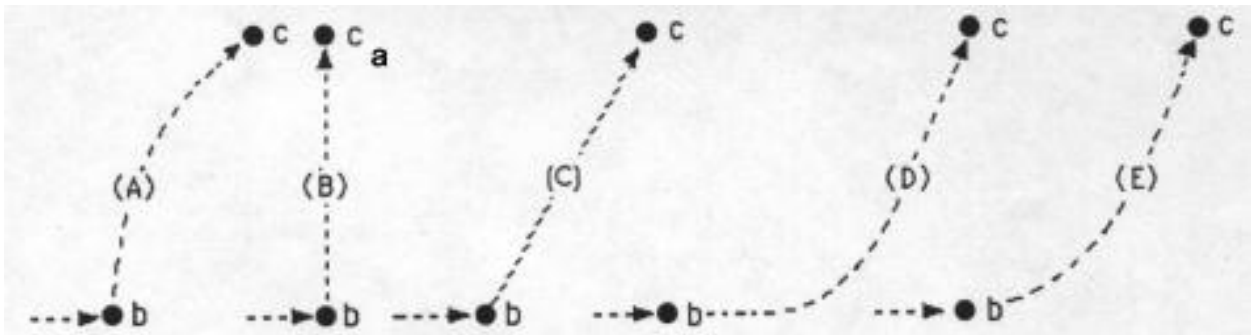
ANS: **E**

The total speed of the puck is the vector sum of both the initial speed and the kick speed. Since the speeds are perpendicular, their vector sum will be less than their arithmetic sum (as according to the Pythagorean theorem).

5. A rocket drifts sideways in outer space from point “a” to point “b” as shown below. The rocket is subject to no outside forces. Starting at position “b”, the rocket’s engine is turned on and produces a constant thrust (force on the rocket) at right angles to the line “ab”. The constant thrust is maintained until the rocket reaches a point “c” in space.<sup>5</sup>



- a) Which of the following best represents the path of the rocket between points “b” and “c”?



ANS: **E**

**B** and **C** describe straight paths which indicate uniform motion over time, but a constant thrust is maintained. **A** describes a slowing down in the vertical direction, or a speeding up in the horizontal direction, and neither scenario is true. **D** and **E** are identical except that there is a slight delay in acceleration in **D**, which does not fit the scenario.

<sup>5</sup> Force Concept Inventory, Hestenes, Halloun, Wells, and Swackhamer, #21, 22, 23, 24

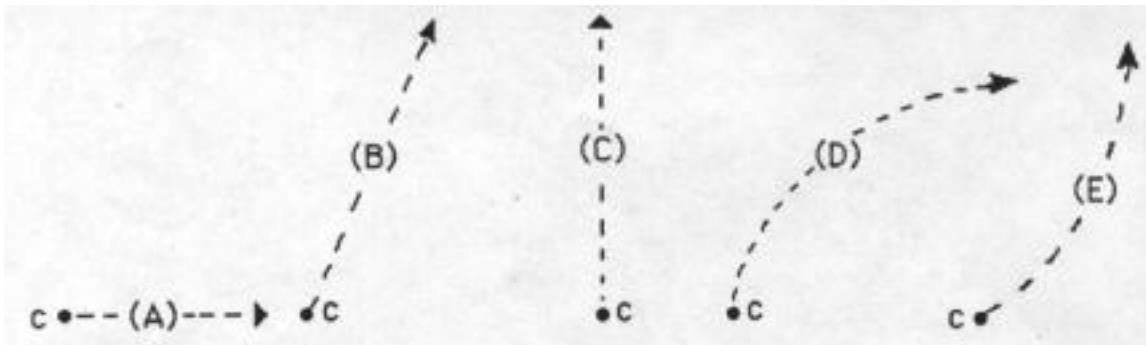


- b) As the rocket moves from position “b” to position “c” its speed is:
- (a) constant.
  - (b) continuously increasing.
  - (c) continuously decreasing.
  - (d) increasing for a while and constant thereafter.
  - (e) constant for a while and decreasing thereafter.

ANS: **B**

Since the thrust is constant during the entire trip from **b** to **c**, the rocket must be accelerating, and in this case, speeding up, since the force and the motion are parallel in the vertical direction.

- c) At point “c” the rocket’s engine is turned off and the thrust immediately drops to zero. Which of the paths below will the rocket follow beyond point “c”?



ANS: **B**

The motion of the rocket the instant before the thrust is removed will persist (it will not change because there is no longer a force acting on it), which matches the path described in **B**.

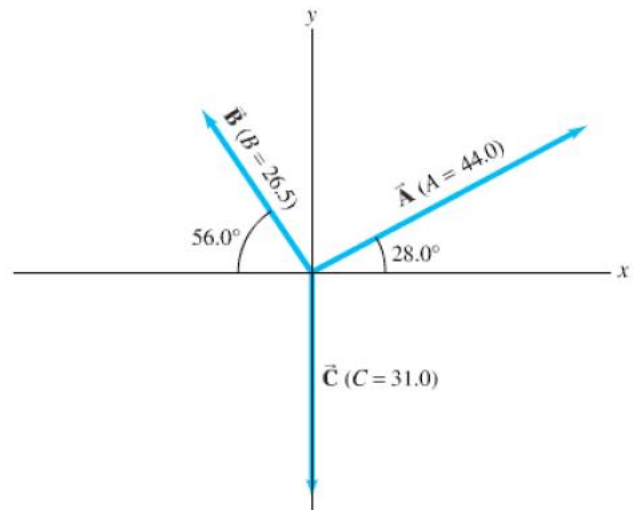
- d) Beyond position “c” the speed of the rocket is:
- (a) constant.
  - (b) continuously increasing.
  - (c) continuously decreasing.
  - (d) increasing for a while and constant thereafter.
  - (e) constant for a while and decreasing thereafter.

ANS: **A**

Since there is no longer any external force acting on the rocket, its motion will be uniform.

**Problems**

6. Three vectors are shown to the right. Their magnitudes are given in arbitrary units.<sup>6</sup>
- a) Determine the sum of the three vectors. Give the resultant in terms of (a) components and (b) magnitude and angle with the x-axis.



<sup>6</sup> Physics 6<sup>th</sup> Edition, Giancoli, Chapter 3 Problems, #10,11,14

$$A_x = 44.0 \cos 28.0^\circ = 38.85 \quad A_y = 44.0 \sin 28.0^\circ = 20.66$$

$$B_x = -26.5 \cos 56.0^\circ = -14.82 \quad B_y = 26.5 \sin 56.0^\circ = 21.97$$

$$C_x = 31.0 \cos 270^\circ = 0.0 \quad C_y = 31.0 \sin 270^\circ = -31.0$$

$$(a) \quad (\vec{A} + \vec{B} + \vec{C})_x = 38.85 + (-14.82) + 0.0 = 24.03 = \boxed{24.0}$$

$$(\vec{A} + \vec{B} + \vec{C})_y = 20.66 + 21.97 + (-31.0) = 11.63 = \boxed{11.6}$$

$$(b) \quad |\vec{A} + \vec{B} + \vec{C}| = \sqrt{(24.03)^2 + (11.63)^2} = \boxed{26.7} \quad \theta = \tan^{-1} \frac{11.63}{24.03} = \boxed{25.8^\circ}$$

b) Determine the vector  $\vec{A} - \vec{C}$ .

$$A_x = 44.0 \cos 28.0^\circ = 38.85 \quad A_y = 44.0 \sin 28.0^\circ = 20.66$$

$$C_x = 31.0 \cos 270^\circ = 0.0 \quad C_y = 31.0 \sin 270^\circ = -31.0$$

$$(\vec{A} - \vec{C})_x = 38.85 - 0.0 = 38.85 \quad (\vec{A} - \vec{C})_y = 20.66 - (-31.0) = 51.66$$

$$|\vec{A} - \vec{C}| = \sqrt{(38.85)^2 + (51.66)^2} = \boxed{64.6} \quad \theta = \tan^{-1} \frac{51.66}{38.85} = \boxed{53.1^\circ}$$

c) Determine the vector (a)  $\vec{B} - 2\vec{A}$  and (b)  $2\vec{A} - 3\vec{B} + 2\vec{C}$ .

$$A_x = 44.0 \cos 28.0^\circ = 38.85 \quad A_y = 44.0 \sin 28.0^\circ = 20.66$$

$$B_x = -26.5 \cos 56.0^\circ = -14.82 \quad B_y = 26.5 \sin 56.0^\circ = 21.97$$

$$C_x = 31.0 \cos 270^\circ = 0.0 \quad C_y = 31.0 \sin 270^\circ = -31.0$$

$$(a) \quad (\vec{B} - 2\vec{A})_x = -14.82 - 2(38.85) = -92.52 \quad (\vec{B} - 2\vec{A})_y = 21.97 - 2(20.66) = -19.35$$

Note that since both components are negative, the vector is in the 3<sup>rd</sup> quadrant.

$$|\vec{B} - 2\vec{A}| = \sqrt{(-92.52)^2 + (-19.35)^2} = \boxed{94.5} \quad \theta = \tan^{-1} \frac{-19.35}{-92.52} = \boxed{11.8^\circ \text{ below } -x \text{ axis}}$$

$$(b) \quad (2\vec{A} - 3\vec{B} + 2\vec{C})_x = 2(38.85) - 3(-14.82) + 2(0.0) = 122.16$$

$$(2\vec{A} - 3\vec{B} + 2\vec{C})_y = 2(20.66) - 3(21.97) + 2(-31.0) = -86.59$$

Note that since the  $x$  component is positive and the  $y$  component is negative, the vector is in the 4<sup>th</sup> quadrant.

$$|2\vec{A} - 3\vec{B} + 2\vec{C}| = \sqrt{(122.16)^2 + (-86.59)^2} = \boxed{149.7} \quad \theta = \tan^{-1} \frac{-86.59}{122.16} = \boxed{35.3^\circ \text{ below } +x \text{ axis}}$$

7. A skier accelerates at a rate of  $4.0 \text{ m/s}^2$  down a ski hill inclined at  $35^\circ$ . What are the vertical and horizontal components of her acceleration?<sup>7</sup>

$$\text{Horizontal: } a_x = (4.0 \text{ m/s}^2) \cos 35^\circ$$

$$a_x = 3.3 \text{ m/s}^2$$

$$\text{Vertical: } a_y = (-4.0 \text{ m/s}^2) \sin 35^\circ$$

$$a_y = -2.3 \text{ m/s}^2$$

<sup>7</sup> Physics Book Two, Irwin Publishing, Chapter 2 Problems, #16

8. A projectile is launched with a horizontal velocity of 10 m/s and a vertical velocity of 20 m/s. What is the magnitude and direction of the projectile's initial velocity?<sup>8</sup>

$$|v_i| = \sqrt{(10 \text{ m/s})^2 + (20 \text{ m/s})^2}$$

$$|v_i| = 22 \text{ m/s}$$

$$\theta = \tan^{-1} \left( \frac{20 \text{ m/s}}{10 \text{ m/s}} \right)$$

$$\theta = 63^\circ$$

Therefore,  $\vec{v}_i = 22 \text{ m/s}$  inclined  $63^\circ$  to the horizontal.

9. A pizza delivery truck drives 2.0 km [W], followed by 3.0 km [W20°N]. What is the total displacement of the delivery truck?<sup>9</sup>

Adding by components:

$$\sum d_x = (-2.0 \text{ km}) - (3.0 \text{ km}) \cos 20^\circ$$

$$\sum \vec{d}_x = 4.8 \text{ km [W]}$$

$$\sum d_y = (3.0 \text{ km}) \sin 20^\circ$$

$$\sum \vec{d}_y = 1.0 \text{ km [N]}$$

$$\sum d = \sqrt{(4.8 \text{ km})^2 + (1.0 \text{ km})^2}$$

$$\sum \vec{d} = 4.9 \text{ km}$$

$$\theta = \tan^{-1} \left( \frac{1.0 \text{ km}}{4.8 \text{ km}} \right)$$

$$\theta = 12^\circ$$

Therefore,  $\vec{d} = 4.9 \text{ km [W}12^\circ\text{N]}$ .

10. In a mall, a shopper rides up an escalator between floors. At the top of the escalator, the shopper turns right and walks 9.00 m to a store. The magnitude of the shopper's displacement from the bottom of the escalator is 16.0 m. The vertical distance between the floors is 6.00 m. At what angle is the escalator inclined above the horizontal?<sup>10</sup>

The escalator is the hypotenuse of a right triangle formed by the lower floor and the vertical distance between floors, as shown in the figure.

The angle  $\theta$  at which the escalator is inclined above the horizontal is related to the length  $L$  of the escalator and the vertical distance between the floors by the sine function:

$$\sin \theta = \frac{6.00 \text{ m}}{L} \quad (1)$$

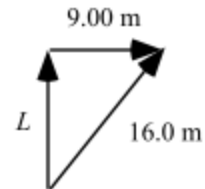
The length  $L$  of the escalator can be found from the right triangle formed by the components of the shopper's displacement up the escalator and to the right of the escalator. From the Pythagorean theorem, we have

$$L^2 + (9.00 \text{ m})^2 = (16.0 \text{ m})^2$$

$$L = \sqrt{(9.00 \text{ m})^2 - (16.0 \text{ m})^2} = 13.2 \text{ m}$$

From equation (1) we have

$$\theta = \sin^{-1} \left( \frac{6.00 \text{ m}}{13.2 \text{ m}} \right) = 27.0^\circ$$



<sup>8</sup> Physics Book Two, Irwin Publishing, Chapter 2 Problems, #18

<sup>9</sup> Physics Book Two, Irwin Publishing, Chapter 2 Problems, #17

<sup>10</sup> Physics, 7<sup>th</sup> Edition, Cutnell & Johnson, Chapter 3 Problems, #9

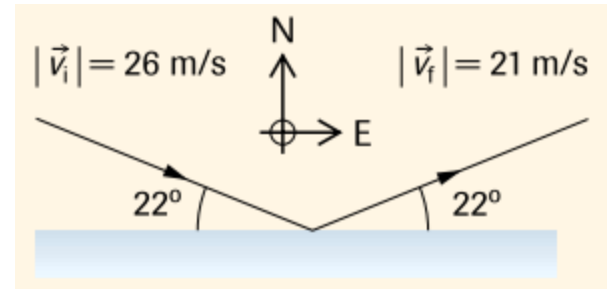
11. A tennis ball's initial velocity is 30 m/s [S]. When struck by a tennis racquet, its velocity becomes 28 m/s [N30°W]. Determine the ball's change in velocity.<sup>11</sup>

$$\begin{aligned}\vec{\Delta v} &= \vec{v}_f - \vec{v}_i \\ \vec{\Delta v} &= 28 \text{ m/s [N30°W]} - 30 \text{ m/s [S]} \\ \vec{\Delta v} &= 28 \text{ m/s [N30°W]} + 30 \text{ m/s [N]} \\ \text{Adding the vectors by components,} \\ \vec{\Delta v} &= 56 \text{ m/s [N15°W]}\end{aligned}$$

12. A billiard ball with an initial velocity of 2.0 m/s [S30°E] strikes the bumper of a billiard table and reflects off it with a velocity of 1.8 m/s [N30°E]. If the interaction with the bumper takes 0.10 s, determine the vector acceleration of the billiard ball.<sup>12</sup>

$$\begin{aligned}\vec{\Delta v} &= \vec{v}_f - \vec{v}_i & \vec{a} &= \frac{\vec{\Delta v}}{t} \\ \vec{\Delta v} &= 1.8 \text{ m/s [N30°E]} - 2.0 \text{ m/s [S30°E]} \\ \vec{\Delta v} &= 1.8 \text{ m/s [N30°E]} + 2.0 \text{ m/s [N30°W]} & \vec{a} &= \frac{3.3 \text{ m/s [N2°W]}}{0.10 \text{ s}} \\ \text{Adding the vectors by components,} \\ \vec{\Delta v} &= 3.3 \text{ m/s [N2°W]} & \vec{a} &= 33 \text{ m/s}^2 \text{ [N2°W]}\end{aligned}$$

13. A hockey puck rebounds from a board as shown below. The puck is in contact with the board for 2.5 ms. Determine the average acceleration of the puck over the interval.<sup>13</sup>



$$\begin{aligned}\Delta v_y &= (21 \text{ m/s}) \sin 22^\circ - (-26 \text{ m/s}) \sin 22^\circ \\ \Delta v_y &= 17.6 \text{ m/s}\end{aligned}$$

$$\begin{aligned}a_{av,y} &= \frac{\Delta v_y}{\Delta t} \\ &= \frac{17.6 \text{ m/s}}{2.5 \times 10^{-3} \text{ s}}\end{aligned}$$

$$a_{av,y} = 7.0 \times 10^3 \text{ m/s}^2$$

$$\vec{v}_i = 26 \text{ m/s [22° S of E]}$$

$$\vec{v}_f = 21 \text{ m/s [22° N of E]}$$

$$\vec{a}_{av} = ?$$

$$\begin{aligned}|\vec{a}_{av}| &= \sqrt{a_{avx}^2 + a_{avy}^2} \\ &= \sqrt{(-1.9 \times 10^3 \text{ m/s}^2)^2 + (7.0 \times 10^3 \text{ m/s}^2)^2}\end{aligned}$$

$$|\vec{a}_{av}| = 7.3 \times 10^3 \text{ m/s}^2$$

$$\tan \theta = \frac{a_{avy}}{a_{avx}}$$

$$\theta = \tan^{-1} \left( \frac{7.0 \times 10^3 \text{ m/s}^2}{1.9 \times 10^3 \text{ m/s}^2} \right)$$

$$\theta = 75^\circ$$

Using +x eastward and +y northward:

$$\Delta v_x = (21 \text{ m/s}) \cos 22^\circ - (26 \text{ m/s}) \cos 22^\circ$$

$$\Delta v_x = -4.6 \text{ m/s}$$

$$\begin{aligned}a_{av,x} &= \frac{\Delta v_x}{\Delta t} \\ &= \frac{-4.6 \text{ m/s}}{2.5 \times 10^{-3} \text{ s}}\end{aligned}$$

$$a_{av,x} = -1.9 \times 10^3 \text{ m/s}^2$$

Thus, the average acceleration of the puck is  $7.3 \times 10^3 \text{ m/s}^2$  [75° N of W].

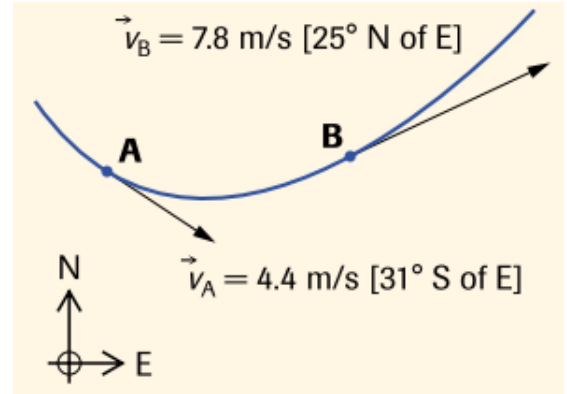
<sup>11</sup> Physics Book Two, Irwin Publishing, Chapter 2 Problems, #20

<sup>12</sup> Physics Book Two, Irwin Publishing, Chapter 2 Problems, #21

<sup>13</sup> Physics 12, Nelson Education, Section 1.2 Practice, #27



14. A bird takes 8.5 s to fly from position A to position B along the path shown in the diagram. Determine the bird's average acceleration.<sup>14</sup>



$$\vec{v}_A = 4.4 \text{ m/s [31° S of E]}$$

$$\vec{v}_B = 7.8 \text{ m/s [25° N of E]}$$

$$\Delta t = 8.5 \text{ s}$$

$$\vec{a} = ?$$

Using +x east and +y north, we find the components of the velocities and the

$$v_{Ax} = 4.4 \text{ m/s} (\cos 31^\circ)$$

$$v_{Bx} = 7.8 \text{ m/s} (\cos 25^\circ)$$

$$v_{Ax} = 3.8 \text{ m/s}$$

$$v_{Bx} = 7.1 \text{ m/s}$$

$$v_{Ay} = -4.4 \text{ m/s} (\sin 31^\circ)$$

$$v_{By} = 7.8 \text{ m/s} (\sin 25^\circ)$$

$$v_{Ay} = -2.3 \text{ m/s}$$

$$v_{By} = 3.3 \text{ m/s}$$

$$a_x = \frac{v_{Bx} - v_{Ax}}{\Delta t}$$

$$= \frac{7.1 \text{ m/s} - 3.8 \text{ m/s}}{8.5 \text{ s}}$$

$$a_y = \frac{v_{By} - v_{Ay}}{\Delta t}$$

$$= \frac{3.3 \text{ m/s} - (-2.3 \text{ m/s})}{8.5 \text{ s}}$$

$$a_x = 0.39 \text{ m/s}^2$$

$$a_y = 0.66 \text{ m/s}^2$$

$$|\vec{a}| = \sqrt{a_x^2 + a_y^2}$$

$$= \sqrt{(0.39 \text{ m/s}^2)^2 + (0.66 \text{ m/s}^2)^2}$$

$$|\vec{a}| = 0.76 \text{ m/s}^2$$

$$\tan \theta = \frac{a_y}{a_x}$$

$$\theta = \tan^{-1} \left( \frac{0.66 \text{ m/s}^2}{0.39 \text{ m/s}^2} \right)$$

$$\theta = 31^\circ$$

The bird's average acceleration is  $0.76 \text{ m/s}^2$  [ $31^\circ$  E of N].

<sup>14</sup> Physics 12, Nelson Education, Section 1.2 Questions, #15